Performance of an Induction Machine given a Torque

Torque: \( \tau_{em} := 243 \text{ N} \cdot \text{m} \)

System Parameters: \( \omega_e := 377.0 \frac{\text{rad}}{\text{sec}} \) \( V_{ll} := 480 \text{ V} \) \( p := 4 \)

\[ V_s := \frac{V_{ll}}{\sqrt{3}} \quad V_s = 277.1281 \text{ V} \]

Machine Parameters:
\( r_s := 0.1 \cdot \text{ohm} \) \( x_{ls} := 0.205 \cdot \text{ohm} \)
\( x_m := 7.15 \cdot \text{ohm} \) \( P_{rot} := 2950 \text{ W} \)
\( r_r := 0.079 \cdot \text{ohm} \) \( x_{lr} := 0.186 \cdot \text{ohm} \)

Precalculations:
\[ V_{th} := V_s \cdot \frac{j \cdot x_m}{r_s + j \cdot (x_{ls} + x_m)} \]
\( V_{th} = 269.3542 + 3.6622i \text{ V} \)
\[ |V_{th}| = 269.3791 \text{ V} \]
\[ Z_{th} := \frac{(r_s + j \cdot x_{ls}) \cdot j \cdot x_m}{r_s + j \cdot (x_{ls} + x_m)} + j \cdot x_{lr} \]
\( Z_{th} = 0.0945 + 0.3866i \cdot \text{ohm} \)
\[ R_{th} := \text{Re}(Z_{th}) \quad X_{th} := \text{Im}(Z_{th}) \]
\( R_{th} = 0.0945 \text{ ohm} \) \( X_{th} = 0.3866 \text{ ohm} \)
\[ X_{thS} := X_{th}^2 \]
\( X_{thS} = 0.1494 \text{ ohm}^2 \)
\[ \omega_{syn} := \frac{\omega_e}{p \cdot \frac{2}{p}} \]
\( \omega_{syn} = 188.5 \text{ rad} \cdot \text{sec}^{-1} \)
\( \tau_{em} = 243 \text{ N} \cdot \text{m} \)

Initial guesses: \( s := 0.02 \)
\[ \tau_{em} = \frac{3 \cdot |V_{th}|^2}{\left[ \left( \frac{r_r}{s} + R_{th} \right)^2 + X_{thS} \right]} \cdot \frac{r_r}{s \cdot \omega_{syn}} \]
\[ s := \text{Find}(s) \]

Result: \( s = 0.01744 \)
\[ \omega_{rm} := (1 - s) \cdot \omega_{syn} \]
\( \omega_{rm} = 185.2117 \frac{\text{rad}}{\text{sec}} \)
\[ \left[ \left( \frac{r_r}{s} + R_{th} \right)^2 + X_{thS} \right] \cdot \frac{r_r}{s \cdot \omega_{syn}} = 243 \text{ N} \cdot \text{m} \]

As a check
Calculations:

\[ z_r := \frac{r_r}{s} + jx_{Ir} \quad \therefore \quad z_r = 4.5287 + 0.186i \text{ ohm} \]

\[ |z_r| = 4.5325 \text{ ohm} \quad \arg(z_r) = 2.3519 \text{ deg} \]

\[ Z_f := \frac{jx_{im}z_r}{z_r + jx_{im}} \quad \therefore \quad Z_f = 3.1149 + 2.1042i \text{ ohm} \]

\[ |Z_f| = 3.759 \text{ ohm} \quad \arg(Z_f) = 34.0398 \text{ deg} \]

\[ R_f := \text{Re}(Z_f) \quad \therefore \quad R_f = 3.1149 \text{ ohm} \]

\[ Z_{\text{in}} := Z_f + r_s + jx_{Is} \quad \therefore \quad Z_{\text{in}} = 3.2149 + 2.3092i \text{ ohm} \]

\[ |Z_{\text{in}}| = 3.9583 \text{ ohm} \quad \arg(Z_{\text{in}}) = 35.6887 \text{ deg} \]

\[ I_s := \frac{V_s}{Z_{\text{in}}} \quad \therefore \quad I_s = 56.8642 - 40.844i \text{ A} \]

\[ |I_s| = 70.0127 \text{ A} \quad \arg(I_s) = -35.6887 \text{ deg} \]

\[ \theta := -\arg(I_s) \quad \therefore \quad \theta = 35.6887 \text{ deg} \]

\[ \text{pf} := \cos(\theta) \quad \therefore \quad \text{pf} = 0.8122 \]

\[ P_{\text{in}} := \sqrt{3} \cdot V_{ll} \cdot |I_s| \cdot \text{pf} \quad \therefore \quad P_{\text{in}} = 47276.0324 \text{ W} \]

\[ P_{\text{loss}} := 3 \cdot (|I_s|)^2 \cdot r_s \quad \therefore \quad P_{\text{loss}} = 1470.5324 \text{ W} \]

\[ P_{\text{gap}} := 3 \cdot (|I_s|)^2 \cdot R_f \quad \therefore \quad P_{\text{gap}} = 45805.5 \text{ W} \]

\[ P_{\text{loss2}} := s \cdot P_{\text{gap}} \quad \therefore \quad P_{\text{loss2}} = 799.052 \text{ W} \]

\[ P_{\text{em}} := (1 - s) \cdot P_{\text{gap}} \quad \therefore \quad P_{\text{em}} = 45006.448 \text{ W} \]

\[ \tau_{\text{em}} := \frac{P_{\text{gap}}}{\omega_{\text{syn}}} \quad \therefore \quad \tau_{\text{em}} = 243 \text{ N-m} \]

\[ P_{\text{out}} := P_{\text{em}} - P_{\text{rot}} \quad \therefore \quad P_{\text{out}} = 42056.448 \text{ W} \]

\[ \tau_{\text{out}} := \frac{P_{\text{out}}}{\omega_{\text{rm}}} \quad \therefore \quad \tau_{\text{out}} = 227.0723 \text{ N-m} \]

\[ \eta := \frac{P_{\text{out}}}{P_{\text{in}}} \quad \therefore \quad \eta = 0.8896 \]