$$j := \sqrt{-1} \quad \text{pu} := 1 \quad \text{MVA} := 1000\,\text{kW}$$

7.11. Three 10 MVA, 100-15 kV transformers have nameplate impedances of 10% and are connected \( \Delta-Y \) with the high voltage side \( \Delta \). Find the zero sequence equivalent circuit.

![Zero sequence equivalent circuit diagram]

Transformer nameplate data:

$$X_{T\_pu} := 0.1\text{pu}$$

$$S_T := 10\text{MVA}$$

$$V_L := 15\text{kV}$$

$$V_H := 100\text{kV}$$

Per-unit Base quantities:

$$S_B := 3 \cdot S_T \quad S_B = 30\text{MVA} \quad \text{Total system complex power}$$

$$V_{BL} := V_L \sqrt{3} \quad V_{BL} = 25.981\text{kV} \quad \text{Line-Line low side base voltage}$$

$$V_{BH} := V_H \quad V_{BH} = 100\text{kV} \quad \text{Line-Line high side base voltage}$$

$$Z_{BL} := \frac{V_{BL}^2}{S_B} \quad Z_{BL} = 22.5\Omega \quad \text{Low side impedance base}$$

$$Z_{BH} := \frac{V_{BH}^2}{S_B} \quad Z_{BH} = 333.333\Omega \quad \text{High side impedance base}$$
The assumed electrical frequency $f_e := 60\text{Hz}$

Generalized Zero Sequence Equivalent Circuit

(a) If the neutral is ungrounded:

$Z_n = \infty$

Zero Sequence Equivalent Circuit for Ungrounded Neutral

(b) If the neutral is grounded solidly:

$Z_{n_{pu}} := 0\text{pu}$

$Z_{0_{pu}} := j \cdot X_T_{pu} + 3 \cdot Z_{n_{pu}}$

$Z_{0_{pu}} = 0.1j \text{pu}$

$|Z_{0_{pu}}| = 0.1\text{pu}$

$\arg(Z_{0_{pu}}) = 90\text{deg}$
(c) If the neutral is grounded through a 5 Ω resistance:

\[ Z_{n_{res}} := 5Ω \]

\[ Z_{n_{res, pu}} := \frac{Z_{n_{res}}}{Z_{BL}} \]

\[ Z_{n_{res, pu}} = 0.222 \text{ pu} \quad 3Z_{n_{res, pu}} = 0.667 \text{ pu} \]

\[ Z_{0_{res, pu}} := j \cdot X_{\Gamma_{pu}} + 3 \cdot Z_{n_{res, pu}} \]

\[ Z_{0_{res, pu}} = 0.667 + 0.1j \text{ pu} \quad |Z_{0_{res, pu}}| = 0.674 \text{ pu} \quad \arg(Z_{0_{res, pu}}) = 8.531 \text{ deg} \]

Zero Sequence Equivalent Circuit for Neutral Grounded through 5 Ω Resistance

(d) If the neutral is grounded through a 5000μF capacitance:

\[ C_{n} := 5000 \mu \text{F} \]

\[ Z_{n_{cap}} := \frac{1}{j \cdot 2 \cdot \pi \cdot f_{e} \cdot C_{n}} \quad C_{n} = 5 \times 10^{-3} \text{ F} \]

\[ Z_{n_{cap}} = -0.531j \Omega \]

\[ Z_{n_{cap, pu}} := \frac{Z_{n_{cap}}}{Z_{BL}} \]

\[ Z_{n_{cap, pu}} = -0.024j \text{ pu} \quad 3Z_{n_{cap, pu}} = -0.071j \text{ pu} \]

\[ Z_{0_{cap, pu}} := j \cdot X_{\Gamma_{pu}} + 3 \cdot Z_{n_{cap, pu}} \]

\[ Z_{0_{cap, pu}} = 0.029j \text{ pu} \quad |Z_{0_{cap, pu}}| = 0.029 \text{ pu} \quad \arg(Z_{0_{cap, pu}}) = 90 \text{ deg} \]
Zero Sequence Equivalent Circuit for Neutral Grounded through $5000 \mu F$ Capacitance
\[ j := \sqrt{-1} \quad \text{pu} := 1 \quad \text{MVA} := 1000.\text{kW} \quad a := e^{\frac{2\cdot\pi}{3}} \]

7.24. In the transformer connection shown below each single-phase transformer has a turns ratio of \( n = n_1/n_2 = 10 \). The low-voltage side carries an unbalanced load with sequence currents given as \( I_{a1} = -I_{a2} = 100 \, @ \, -30^\circ \text{A} \). Find the sequence currents \( I_{A1} \) and \( I_{A2} \) as phasors.

From the connection shown above, we have:

\[ I_a = n \cdot I_A - n \cdot I_C \quad \text{where:} \quad n = \frac{n_1}{n_2} \]

\[ I_a = n \cdot (I_A - I_C) \quad n := \frac{10}{1} \]

Writing the above equation for positive sequence quantities (loosely following the development in Blackburn):

\[ I_{a1} = n \cdot (I_{A1} - I_{C1}) \]

and noting that
\[ I_{C1} = a \cdot I_{A1} \]

Therefore:

\[ I_{a1} = n \left( I_{A1} - a \cdot I_{A1} \right) \]

\[ I_{a1} = n \cdot (1 - a) \cdot I_{A1} \]

We note: \( 1 - a = 1.5 - 0.866i \)

\[ |1 - a| = 1.732 \]

\[ \text{arg}(1 - a) = -30 \text{ deg} \]

Thus:

\[ I_{a1} = n \sqrt{3} \cdot I_{A1} e^{-j30\deg} \]

or:

\[ I_{A1} = \frac{I_{a1} e^{j30\deg}}{n \sqrt{3}} \]

For the given positive sequence unbalanced load current:

\[ I_{a1} := 100 e^{-j30\deg} \]

\[ I_{A1} := \frac{I_{a1}}{n \cdot (1 - a)} \]

\[ I_{A1} = 5.774 \quad |I_{A1}| = 5.774 \quad \text{arg}(I_{A1}) = 0 \text{ deg} \]
Writing similar equation for the negative sequence quantities:

\[ I_{a2} = n(I_{A2} - I_{C2}) \]

and noting that

\[ I_{C2} = a^2 I_{A2} \]

Therefore:

\[ I_{a2} = n(1 - a^2) I_{A2} \]

We note:

\[ 1 - a^2 = 1.5 + 0.866i \]

\[ |1 - a^2| = 1.732 \]

\[ \arg(1 - a^2) = 30 \text{ deg} \]

Thus:

\[ I_{a2} = n\sqrt{3} I_{A2} e^{j30\text{deg}} \]

or:

\[ I_{A2} = \frac{I_{a2} e^{-j30\text{deg}}}{n\sqrt{3}} \]

For the given positive sequence unbalanced load current:

\[ I_{a2} := -100e^{-j30\text{deg}} \]

\[ I_{A2} := \frac{I_{a2}}{n(1 - a^2)} \]

\[ I_{A2} = -2.887 + 5i \]

\[ |I_{A2}| = 5.774 \]

\[ \arg(I_{A2}) = 120 \text{ deg} \]
As an aid in visualizing the sequence currents on each side of the transformer bank and the impact of the \( \Delta-Y \) connection (the 1/10 for \( I_{a1} \) and \( I_{a2} \) is to scale the quantities for plotting):

Noodling (as in thinking about with one’s noodle (head), not cat-fishing by hand) a bit more with this problem, I thought it would be mildly interesting to look at the phase currents resulting from these sequence currents. Using the synthesis equation here is what I came up with:

\[
A_{012} := \begin{pmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{pmatrix}
\]

\[
\begin{pmatrix}
I_a \\
I_b \\
I_c
\end{pmatrix} := A_{012} \begin{pmatrix}
0 \\
I_{a1} \\
I_{a2}
\end{pmatrix}
\]

\[
\begin{align*}
I_a &= 0 \\
I_b &= -86.603 - 150i \\
I_c &= 86.603 + 150i
\end{align*}
\]
\[ |I_b| = 173.205 \quad \text{arg}(I_b) = -120 \, \text{deg} \]

\[ |I_c| = 173.205 \quad \text{arg}(I_c) = 60 \, \text{deg} \]

It looks like the load is between only phases "b" and "c," and not involving "a" phase. The phase currents on the high-voltage side of the transformer:

\[
\begin{pmatrix}
I_A \\
I_B \\
I_C
\end{pmatrix} := A_{012}
\begin{pmatrix}
0 \\
I_{A1} \\
I_{A2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
I_A \\
I_B \\
I_C
\end{pmatrix} = \begin{pmatrix}
2.887 + 5i \\
-5.774 - 10i \\
2.887 + 5i
\end{pmatrix}
\]

\[
\begin{pmatrix}
I_A \\
I_B \\
I_C
\end{pmatrix} \rightarrow \begin{pmatrix}
5.774 \\
11.547 \\
5.774
\end{pmatrix}
\]

\[
\text{arg} \begin{pmatrix}
I_A \\
I_B \\
I_C
\end{pmatrix} = \begin{pmatrix}
60 \\
-120 \\
60
\end{pmatrix} \, \text{deg}
\]

The result on the high-voltage side of the transformer is a bit more challenging to classify. We have the phase "A" and "C" currents in-phase and equaling the phase "B" current; but at 180 degrees opposite. It's tempting to call it a very "ugly" and "unbalanced" 3-phase fault, definitely not bolted. This problem once again demonstrating the impact of \( \Delta - Y \) transformation on sequence currents and the resulting phase currents.