**Goals for Lecture 11:**

1. Review some basic material to help transition from Joe's style to Andy's.
2. Transition to more of a system block diagram; connecting physical significance to individual components of the diagram.
3. Transformer voltage: develop a block diagram and simulink representation of an inductor and discuss concepts below. Evaluate step changes to the input.
4. ~~Transformer voltage: develop a block diagram and simulink representation of a two-winding transformer. Look at step changes in the input and output.~~
5. ~~Transformer voltage: Look at self and mutual inductances. Show that the transformer goes to an ideal model if leakage is ignored.~~
6. Saturation: Saturation impacts some gain block but not others
7. Introduce saturation model Se.

**Concepts for Lecture 11:**

1. Flux linkages do not change instantly
2. Voltages drive flux linkages and currents develop to support flux linkages
3. Inductance: Self versus mutual
4. Saturation impacts on inductances
5. Time constant is inverse of loop gain (in first order system)

magnetic field intensity (A/m)

**Review from Andy's perspective**

Start with Ampère's Law:

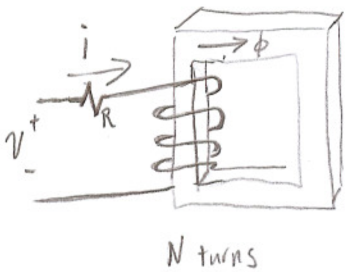
$$\oint_C \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}$$

Current Density (A/m$^2$)

Displacement current

<u>Inductor</u>

※1 assumption: We can ignore displacement current




N turns

$$\oint_c \vec{H}\cdot dl = \int_S \vec{J}\cdot d\vec{s}$$

Integrate in core of inductor


$$Hl = Ni$$

effective length through core

Inductor Continued

$B = \mu H$ -or- $H = \frac{B}{\mu}$

Permeability (H/m)
* a property of the material
* is a function of H
* Sometimes $\mu = \mu_r \mu_o$

Permitlivity of free space (H/m)

Relative permeability (No unit)

magnetic flux density (T -or- AH)

* $\mu_r$ is about 4000 for transformer iron

$\phi = AB$ $\therefore$ $B = \phi / A$

Cross sectional area of core

Substitute $\phi/A$ for B in $H = \frac{B}{\mu}$

$$H = \frac{\phi}{\mu A}$$

Substitute for H in equation $Hl = NI$

$$\frac{\phi l}{\mu A} = Ni$$

Permeance (H/turns)

Let $\mathcal{R} = \frac{l}{\mu A}$ or $\boxed{\mathcal{P} = \frac{1}{\mathcal{R}} = \frac{\mu A}{l}}$

$$\therefore \frac{\phi}{\mathcal{P}} = Ni$$

$$\boxed{\therefore \phi = N\mathcal{P}I}$$

Reluctance (turns/H)

As material saturates $\mu \downarrow$ ; $\mathcal{R} \uparrow$ ; $\mathcal{P} \downarrow$

Inductor continued

Introduce flux linkage $\psi$

$$\psi = N\phi$$

Substitute $NPI$ for $\phi$ yields

$$\psi = N^2 P i$$

As material saturates $\mu\downarrow\ P\downarrow\ L\downarrow$

Define Inductance

Let $L = N^2 P$ or $L = \frac{\psi}{i}$ $\therefore$ $\boxed{i = \psi/L}$

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Faraday's Law

$$e = \frac{d\psi}{dt}$$

$$= \dot{\psi}$$

Eq 3.6 in Kundar

Lenz's law - The emf produced in an electric circuit always acts in such a direction that the current it drives around a closed circuit produces a magetic field which opposes the magnetic flux.

Circuit equation

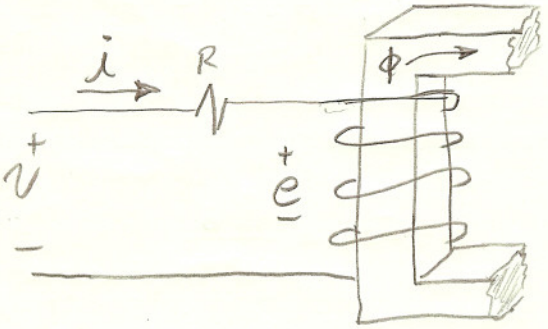



Figure 3.10 in Kundar has wrong direction on $\phi$!

$$v = iR + e$$

$$= iR + \dot{\psi}$$

Eq 3.7 Kundar

$$\boxed{\therefore \dot{\psi} = v - iR}$$

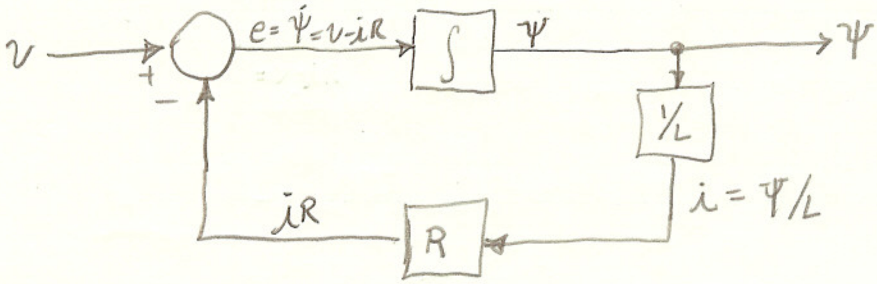
Use the following equations to build a block diagram

$$e = \dot{\psi} = v - iR$$

$$i = \psi/L$$

L13

Inductor Continued




Note: $\Psi$ can not change instantly w/ finite 'e'!

$v$ drives the value of $\Psi$

$i$ is determined by $\Psi$

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Initial conditions in steady state (ss)

$\dot{\Psi}_0 = 0$

$\therefore v_0 - i_0 R = 0$

$\therefore v_0 = i_0 R$      {Kirkoff will be relieved!}

$\therefore i_0 = v/R$

$\Psi_0 = i_0 L$

$= \frac{v_0 L}{R}$

$\therefore \Psi_0 = \frac{v_0 L}{R}$

Also Note:

Loop gain $= \frac{R}{L}$ $\therefore$ Time constant $\tau = \frac{L}{R}$

multiply by special form of 1

As material saturates $\mu\downarrow$ $P\downarrow$ $L\downarrow$ & $\tau\downarrow$

$\tau = \frac{i_0^2}{i_0^2} \frac{L}{R}$

$= \frac{i^2 L}{i_0 v_0}$      $(v_0 = i_0 R_0)$

Twice the energy stored divided by the rate energy is being dissipated

$= \frac{2E_0}{P_0}$      $(E_0 = \frac{1}{2} i_0 L$ ; $P_0 = v_0 i_0)$

L13