Basic AC Circuit Analysis

**Instantaneous Quantities:**

\[ v(t) = V_m \cos(\omega t + \phi) \]

- Instantaneous quantities will be written using lower case letters
- \( V_m \) is the peak amplitude (from zero to the peak of the sinusoid)
- \( \phi \) is the phase angle
- \( \omega \) is the angular frequency, it has units of rad/sec, but we usually write \( \phi \) in degrees
  - Be careful when performing calculations!
- \( 2\pi \) radians equivalent to 360 degrees (1 full cycle of the sinusoid)
- \( \omega = 2\pi f \) (\( f \) is the frequency in Hz)
- \( T = 1/f \) (period for 1 cycle).

**RMS Value**

- We are normally interested in steady-state behavior of a circuit
- We might keep track of the peak of the cosine wave, but it is often better to use another measurement. Since the average value of a sinusoidal waveform is zero, something else is needed.
- The room mean square (RMS) value is most commonly used.

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \, dt} \]

e.g.

\[ v(t) := 170 \cdot \sin(2\pi \cdot 60 \cdot t) \cdot V \]

\[ T := \frac{1}{60} \quad V_{\text{rms}} := \sqrt{\int_0^T v(t)^2 \, dt} \]

\[ V_{\text{rms}} = 120.208 \text{ V} \]

- For a pure sinusoidal waveform:

\[ V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \]

e.g.

\[ V_m := 170 \quad V_{\text{rms}} := \frac{V_m}{\sqrt{2}} \]

\[ V_{\text{rms}} = 120.208 \]
Phasor Analysis

- To describe steady-state behavior we use a phasors. A phasor has a magnitude (RMS) and an angle.

- Phasors will be defined based on the cosine function in this class.

\[ V = |V| \cdot e^{j\phi} \quad j := \sqrt{-1} \quad \text{VAR} := W \]

\[ j\omega L \]

\[ V := 100 \cdot e^{j0 \cdot \text{deg}} \quad L := 300 \cdot \text{mH} \quad R := 200 \cdot \text{ohm} \quad \omega := 2 \cdot \pi \cdot 60 \frac{\text{rad}}{\text{sec}} \]

Node equation:

\[ \frac{V_o - V_i}{j\omega L} + \frac{V_o}{R} = 0 \quad \text{Solve for } V_o \]

Loop equation

\[ -V_i + j\omega L \cdot I + R \cdot I = 0 \quad \text{Solve for } I \]

\[ I := \frac{V}{R + j\omega L} \quad I = 0.379 - 0.214j \quad |I| = 0.435 \text{A} \quad \text{arg}(I) = -29.488 \text{deg} \]

AC Power

Referencing the same circuit

\[ P := (|I|)^2 \cdot R \quad P = 37.885 \text{W} \]

\[ Q := (|I|)^2 (\omega \cdot L) \quad Q = 21.424 \text{VAR} \]

\[ S := P + jQ \quad |S| = 43.523 \text{W} \]

\[ \text{pf} := \frac{P}{|S|} \quad \text{pf} = 0.87 \quad \text{lagging} \]