ECE 320: Lecture 2

Notes

Basic AC Circuit Analysis (Continued)

\[ v(t) = V_m \cos(\omega \cdot t + \phi) \text{ Volts} \]

(Note, units named for a person always start with capital letters).

\[ \vec{V} = |V| \cdot e^{j \cdot \phi} \text{ Volts} \quad \text{where} \quad |V| = \frac{V_m}{\sqrt{2}} \text{ RMS Phasor} \]

- RMS phasor written using upper case, instaneous time quantity lower case.
- The frequency is assumed in this notation.
- The angle of voltage source is the angle relative to a reference point which is given the angle of 0 degrees (or radians).
- This is especially important when there are a large number of voltage sources in a system. In a large power system there might be hundreds of generators to represent.

Response of AC Circuit

- Now lets connect this voltage source to a series R-L circuit. R-L circuits are the most common type. Any conductor or wire has inductance, enough to impact the circuit behavior.
- Also, every conductor has parasitic capacitance relative to some reference, how at 60Hz, the capacitance has a small impact on circuit response unless the conductor is very long.
- We'll look at cases where discrete capacitors are added intentionally.
- The differential equation for this circuit is:

\[ v(t) = R \cdot i(t) + L \cdot \frac{d}{dt} i(t) \]

- The resulting current when the source is switched into the circuit is:

\[ i(t) = \left[ \frac{-V_m}{\sqrt{R^2 + (\omega \cdot L)^2}} \right] \cdot \cos(\phi - \theta) \cdot e^{-\left(\frac{R}{L}\right) \cdot t} + \left[ \frac{V_m}{\sqrt{R^2 + (\omega \cdot L)^2}} \right] \cdot \cos(\omega \cdot t + \phi - \theta) \text{ Amps} \]

- The first term is a decaying dc offset. The initial amplitude of this offset depends on the angle where the switch is closed. This is a transient term
- The second term is the steady-state component. This is our primary concern in this class.
- Not that we have an additional term in the angle, which describes the R-L circuit:

\[ \theta = \tan \left( \frac{\omega \cdot L}{R} \right) \]
• The angle of the current is now: $\phi - \theta$
• If we only look at the sinusoidal steady-state component, we can compute a RMS magnitude

$$|I| = \left[ \frac{V_m}{\sqrt{2}} \cdot \cos(\omega \cdot t + \phi - \theta) \right] \text{ Amps}$$

$$I = |I| \cdot e^{j(\phi - \theta)} \text{ Amps}$$

• We define Impedance as:

$$Z = \sqrt{R^2 + (\omega \cdot L)^2} \cdot e^{j\theta} \text{ } \Omega$$

• This is more commonly written as:

$$Z = |Z| \cdot e^{j\theta} \text{ } \Omega$$

• Note that frequency is still assumed here, but we no longer have a time varying function
• We can also describe an admittance as:

$$Y = \frac{1}{Z} = \frac{|Y| \cdot e^{-j\theta}}{Z} \text{ Mhos or Siemens}$$

• Where

$$Z = R + j \cdot X \text{ Resistance and Reactance}$$

$$Y = G + j \cdot B \text{ Conductance and Susceptance}$$

• In general we can't say that $G = 1/R$ or $B = 1/X$ (unless the $X$ or $R$ terms are zero respectively)
• For an pure inductor ($R = 0$). Note that pure inductors and pure resistors don't really exist, every circuit will have at least some $R$ and some $L$, but for now neglect the $R$

$$X = \omega \cdot L \text{ } \text{ } \text{ } B = \frac{1}{(\omega \cdot L)}$$
• For an pure capacitor (R = 0). Again, most capacitors have some small series resistance, but for now neglect the R (which is done in many cases):

\[ X = \frac{1}{\omega \cdot C} \quad B = \omega \cdot C \]

• Now lets look at impedance angles. For a pure resistance (X = 0)

\[ \theta = 0 \]

• For a pure inductance:

\[ \theta = 90 \text{deg} \]

• For a pure capacitance:

\[ \theta = -90 \text{deg} \]

• In general, most circuit are combination, so the angle of the impedance will somewhere in the range:

\[ 90 \text{deg} \leq \theta \leq 90 \text{deg} \]

• Starting from:

\[ I = \frac{V}{Z} \]

• We can define the **Power Factor Angle** as the angle between the voltage and the current.

\[ \text{pf\_angle} = \phi_v - \phi_i \]

• For the simple configuration described above we would have:

\[ \text{pf\_angle} = \theta \]

• We may also use this to define the effective angle computed from measured voltages and currents.

• Just as with the impedance angle we will have:

\[ 90 \text{deg} \leq \text{pf\_angle} \leq 90 \text{deg} \]

unless there is a polarity problem in the measurements.
We can also define the **Power Factor** as

\[ \text{pf} = \cos(\text{pf\_angle}) \]

Where (as a result of the limits on the pf\_angle):

\[ 0 \leq \text{pf} \leq 1.0 \]

Because of this, when one writes (or types) the value for a power factor, they also need to indicate whether the power factor is leading or lagging.

- A lagging power factor means "I lags V". This is common with an R-L circuit. This means that the zero crossing for the current waveform appears after (to the right) the current zero for the voltage.
- A leading power factor means "I leads V". This is common with a series R-C circuit. This means that the zero crossing for the current waveform appears before (to the left) the current zero for the voltage.

**Next time:**

We will discuss methods for calculating power.