ECE 523: Lecture 9

$$a := e^{j\cdot 120deg} \qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Case 1:

$Zaa := (14.9 + j\cdot 58.4)ohm$ $\qquad Zab := (4 + j\cdot 27.3)ohm$

$Zbb := (14.9 + j\cdot 58.4)ohm$ $\qquad Zac := (4 + j\cdot 27.3)ohm$

$Zcc := (14.9 + j\cdot 58.4)ohm$ $\qquad Zbc := (4 + j\cdot 27.3)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1}\cdot Z_{ABC}\cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 10.9 + 31.1i \end{pmatrix} \Omega$$

$$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc)$$

$$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac)$$

$Z_s - Z_m = 10.9 + 31.1i\Omega$ $\qquad Z_s - Z_m - Z012_{1,1} = 0\Omega$

$Z_s + 2Z_m = 22.9 + 113i\Omega$ $\qquad Z_s + 2Z_m - Z012_{0,0} = 0\Omega$

$Z_1 := Z012_{1,1}$ $Z_1 = 10.9 + 31.1i\Omega$

$Z_2 := Z012_{2,2}$ $Z_2 = 10.9 + 31.1i\Omega$

$Z_0 := Z012_{0,0}$ $Z_0 = 22.9 + 113i\Omega$

Case 2: (now add imbalance to self terms--act as if it is a similar tower structure with mutual coupling to other conductors creating unbalance, although I'm being somewhat unrealistic for comparison purposes)

$Zaa := (14.9 + j\cdot 53.4)ohm$ $Zab := (4 + j\cdot 27.3)ohm$

$Zbb := (14.9 + j\cdot 68.4)ohm$ $Zac := (4 + j\cdot 27.3)ohm$

$Zcc := (14.9 + j\cdot 53.4)ohm$ $Zbc := (4 + j\cdot 27.3)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$Z_{012} := A_{012}^{-1}\cdot Z_{ABC}\cdot A_{012}$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & 4.33 - 2.5i & -4.33 - 2.5i \\ -4.33 - 2.5i & 10.9 + 31.1i & 4.33 - 2.5i \\ 4.33 - 2.5i & -4.33 - 2.5i & 10.9 + 31.1i \end{pmatrix}\Omega$$

$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc)$ $Z_s = 14.9 + 58.4i\Omega$

$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac)$ $Z_m = 4 + 27.3i\Omega$

$Z_s - Z_m = 10.9 + 31.1i\Omega$ $Z_s - Z_m - Z012_{1,1} = 0\Omega$

$Z_s + 2Z_m = 22.9 + 113i\Omega$ $Z_s + 2Z_m - Z012_{0,0} = 0\Omega$

Compare to original case:

$Z_0 - (Z_s + 2Z_m) = 0\Omega$ $Z_1 - (Z_s - Z_m) = 0\Omega$

Case 3: (A more realistic set of unbalances, interaction between a line and static wires- for now the changes in the mutual terms is not included)

$Zaa := (14.9 + j\cdot 57.0)ohm$ $Zab := (4 + j\cdot 27.3)ohm$

$Zbb := (14.9 + j\cdot 58.2)ohm$ $Zac := (4 + j\cdot 27.3)ohm$

$Zcc := (14.9 + j\cdot 60.0)ohm$ $Zbc := (4 + j\cdot 27.3)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1}\cdot Z_{ABC}\cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & -0.52 - 0.7i & 0.52 - 0.7i \\ 0.52 - 0.7i & 10.9 + 31.1i & -0.52 - 0.7i \\ -0.52 - 0.7i & 0.52 - 0.7i & 10.9 + 31.1i \end{pmatrix}\Omega$$

$Z_s := \frac{1}{3}(Zaa + Zbb + Zcc)$ $Z_s = 14.9 + 58.4i\Omega$

$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac)$ $Z_m = 4 + 27.3i\Omega$

$Z_s - Z_m = 10.9 + 31.1i\Omega$ $Z_s - Z_m - Z012_{1,1} = 0\Omega$

$Z_s + 2Z_m = 22.9 + 113i\Omega$ $Z_s + 2Z_m - Z012_{0,0} = 0\Omega$

Compare to original case:

$Z_0 - (Z_s + 2Z_m) = 0\Omega$ $Z_1 - (Z_s - Z_m) = 0\Omega$

Case 4: (Now add imbalances to mutual terms)

$Zaa := (14.9 + j\cdot 58.4)ohm$ $Zab := (4 + j\cdot 27.4)ohm$

$Zbb := (14.9 + j\cdot 58.4)ohm$ $Zac := (4 + j\cdot 28.0)ohm$

$Zcc := (14.9 + j\cdot 58.4)ohm$ $Zbc := (4 + j\cdot 26.5)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1}\cdot Z_{ABC}\cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & -0.17 + 0.4i & 0.17 + 0.4i \\ 0.17 + 0.4i & 10.9 + 31.1i & 0.35 - 0.8i \\ -0.17 + 0.4i & -0.35 - 0.8i & 10.9 + 31.1i \end{pmatrix} \Omega$$

$Z_{s2} := \frac{1}{3}(Zaa + Zbb + Zcc)$ $Z_{s2} = 14.9 + 58.4i\Omega$ $Z_s = 14.9 + 58.4i\Omega$

$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac)$ $Z_m = 4 + 27.3i\Omega$

$Z_s - Z_m = 10.9 + 31.1i\Omega$ $Z_s - Z_m - Z_{012_{1,1}} = 0\Omega$

$Z_s + 2Z_m = 22.9 + 113i\Omega$ $Z_s + 2Z_m - Z_{012_{0,0}} = 0\Omega$

Compare to original case:

$Z_0 - (Z_s + 2Z_m) = 0\Omega$ $Z_1 - (Z_s - Z_m) = 0\Omega$

Case 5 (both self and mutual unbalanced):

$Zaa := (14.9 + j\cdot 57.0)ohm$ $Zab := (4 + j\cdot 27.4)ohm$

$Zbb := (14.9 + j\cdot 58.2)ohm$ $Zac := (4 + j\cdot 28.0)ohm$

$Zcc := (14.9 + j\cdot 60.0)ohm$ $Zbc := (4 + j\cdot 26.5)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1}\cdot Z_{ABC}\cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.9 + 113i & -0.69 - 0.3i & 0.69 - 0.3i \\ 0.69 - 0.3i & 10.9 + 31.1i & -0.17 - 1.5i \\ -0.69 - 0.3i & 0.17 - 1.5i & 10.9 + 31.1i \end{pmatrix} \Omega$$

$Z_{s2} := \frac{1}{3}(Zaa + Zbb + Zcc)$ $Z_{s2} = 14.9 + 58.4i\Omega$ $Z_s = 14.9 + 58.4i\Omega$

$Z_m := \frac{1}{3}\cdot(Zab + Zbc + Zac)$ $Z_m = 4 + 27.3i\Omega$

$Z_s - Z_m = 10.9 + 31.1i\Omega$ $Z_s - Z_m - Z_{012_{1,1}} = 0\Omega$

$Z_s + 2Z_m = 22.9 + 113i\Omega$ $Z_s + 2Z_m - Z_{012_{0,0}} = 0\Omega$

Compare to original case:

$Z_0 - \left(Z_s + 2Z_m\right) = 0\Omega$ $Z_1 - \left(Z_s - Z_m\right) = 0\Omega$