ECE 523: Lecture 6

An unbalanced n-phase set of phasors can be represented by n-1 balanced n-phase sets of phasors and a zero phase set of phasors all added together by superposition.

$$V_a = V_{a1} + V_{a2} + V_{a3} .. V_{an}$$

$$V_b = V_{b1} + V_{b2} + V_{b3} .. V_{bn}$$

etc.

$$V_n = V_{n1} + V_{n2} + V_{n3} .. V_{n,n-1}$$

- Define a phase angle shift term a:

$a = e^{j \cdot \frac{2 \cdot \pi}{n}}$ Note for a three phase system: $a := e^{j \cdot \frac{2 \cdot \pi}{3}}$ $\arg(a) = 120\,\text{deg}$

- Now make a into a function that depends on n. Leave n as a variable we can define. For now set a value

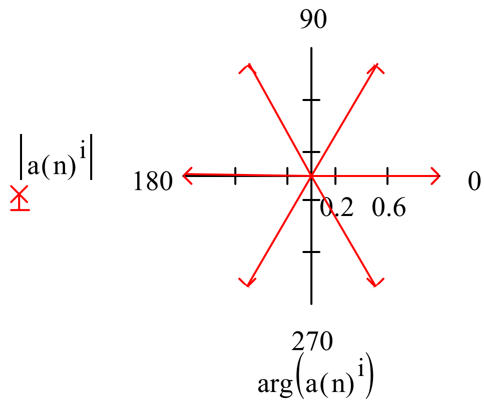
$n := 6$

$$a(n) := 1e^{j \cdot \frac{2 \cdot \pi}{n}}$$

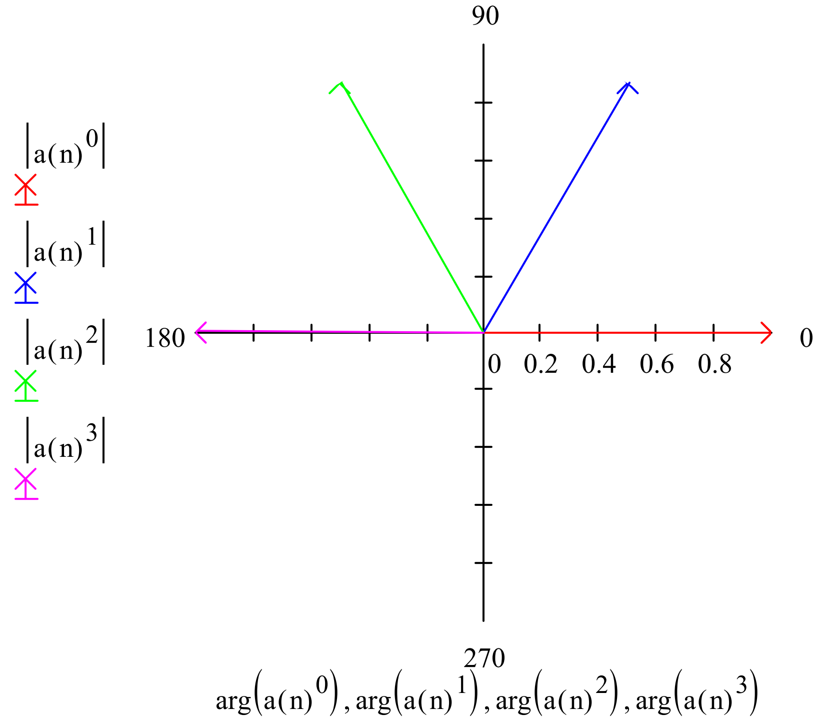
We can also define an array index, and have terms that vary as this increments.

$$i := 0, 1 .. (n-1)$$

- Polar plot, with magnitude anangle as "i" increments
- Set line type to "stem" in the properties
- Note that we can't tell which way it rotates as "i" increments



This time we will actually show the powers of "a"



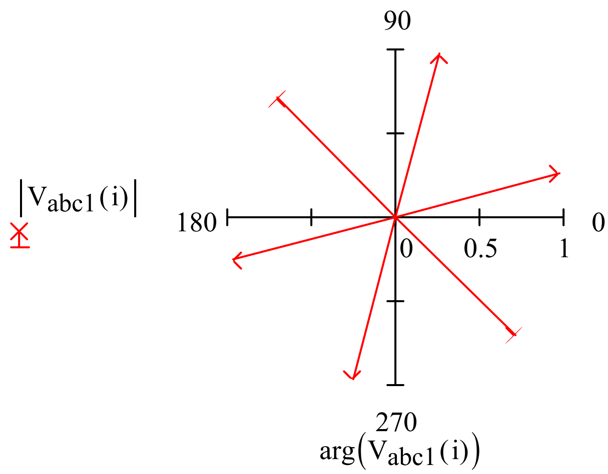
Now look at the balanced n-phase sets:

- Define an initial magnitude and angle reference:

$$V_{1ref} := 1.0e^{j\cdot 15deg}$$

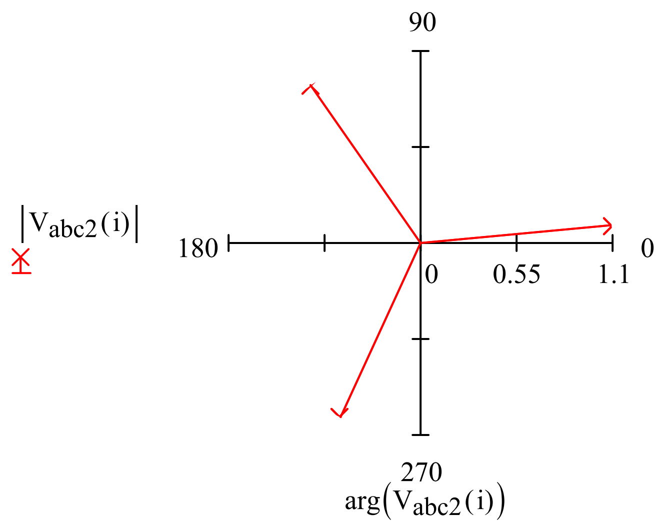
- Phase relationship for phase sequence 1:

$$V_{abc1}(i) := a(n)^{n-i}\cdot V_{1ref}$$



- Phase relationship for phase sequence 2:

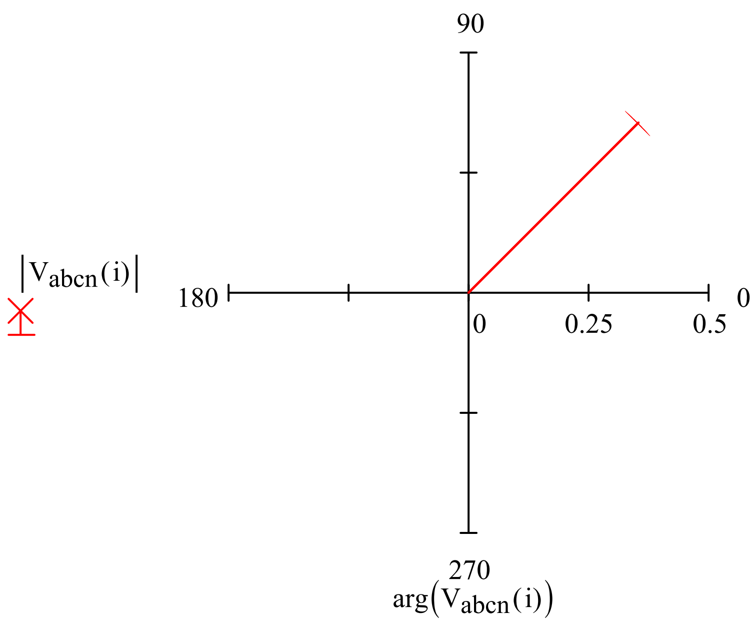
$V_{2ref} := 1.1e^{j\cdot 5deg}$ $\qquad V_{abc2}(i) := a(n)^{2(n-i)}\cdot V_{2ref}$



- Relationship for phase sequence "n"

$V_{nref} := 0.5e^{j\cdot 45deg}$ $\qquad V_{abcn}(i) := a(n)^{n(n-i)}\cdot V_{nref}$

- Try entering different values of n



Sums of columns (to check relationships from lecture 5)

- First the zero column:

$$Col\_0 := \sum_{i=1}^{n} V_{abcn}(i) \qquad |Col\_0| = 3 \qquad arg(Col\_0) = 45\,deg$$

$$\frac{|Col\_0|}{n} = 0.5$$

$$\left(\frac{|Col\_0|}{n}\right) - |V_{nref}| = 0$$

- Now the "1" column (multiply terms by a^i):

$$Col\_1 := \sum_{i=1}^{n} \left(a(n)^{i} \cdot V_{abc1}(i)\right) \qquad |Col\_1| = 6 \qquad arg(Col\_1) = 15\,deg$$

$$\frac{|Col\_1|}{n} = 1$$

Repeat same multiplier on column 2 and column 0

$$Col\_2_1 := \sum_{i=1}^{n} \left(a(n)^{i} \cdot V_{abc2}(i)\right) \qquad |Col\_2_1| = 0$$

$$Col\_0_1 := \sum_{i=1}^{n} \left(a(n)^{i} \cdot V_{abcn}(i)\right) \qquad |Col\_0_1| = 0$$

- Now the "2" column (multiply terms by a^{2i}):

$$Col\_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abc2}(i)\right) \qquad |Col\_2| = 6.6 \qquad arg(Col\_2) = 5\,deg$$

$$\left(\frac{|Col\_2|}{n}\right) - |V_{2ref}| = 0$$

Repeat same multiplier on column 1 and column 0

$$Col\_1_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abc1}(i) \right) \qquad \left| Col\_1_2 \right| = 0$$

$$Col\_0_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abcn}(i) \right) \qquad \left| Col\_0_2 \right| = 0$$

Matrix relation (for n = 6)

$$C_{012345} := \frac{1}{6} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a(6) & a(6)^2 & a(6)^3 & a(6)^4 & a(6)^5 \\ 1 & a(6)^2 & a(6)^4 & a(6)^6 & a(6)^8 & a(6)^{10} \\ 1 & a(6)^3 & a(6)^6 & a(6)^9 & a(6)^{12} & a(6)^{15} \\ 1 & a(6)^4 & a(6)^8 & a(6)^{12} & a(6)^{16} & a(6)^{20} \\ 1 & a(6)^5 & a(6)^{10} & a(6)^{15} & a(6)^{20} & a(6)^{25} \end{pmatrix}$$

$$6C_{012345} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5 + 0.87i & -0.5 + 0.87i & -1 & -0.5 - 0.87i & 0.5 - 0.87i \\ 1 & -0.5 + 0.87i & -0.5 - 0.87i & 1 & -0.5 + 0.87i & -0.5 - 0.87i \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5 - 0.87i & -0.5 + 0.87i & 1 & -0.5 - 0.87i & -0.5 + 0.87i \\ 1 & 0.5 - 0.87i & -0.5 - 0.87i & -1 & -0.5 + 0.87i & 0.5 + 0.87i \end{pmatrix}$$

$$C_{012345}^{-1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5 - 0.87i & -0.5 - 0.87i & -1 & -0.5 + 0.87i & 0.5 + 0.87i \\ 1 & -0.5 - 0.87i & -0.5 + 0.87i & 1 & -0.5 - 0.87i & -0.5 + 0.87i \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5 + 0.87i & -0.5 - 0.87i & 1 & -0.5 + 0.87i & -0.5 - 0.87i \\ 1 & 0.5 + 0.87i & -0.5 + 0.87i & -1 & -0.5 - 0.87i & 0.5 - 0.87i \end{pmatrix}$$

Note that:

$a(6) = 0.5 + 0.87i$

$a(6)^2 = -0.5 + 0.87i$

$a(6)^3 = -1$

$a(6)^4 = -0.5 - 0.87i$

$a(6)^5 = 0.5 - 0.87i$

$a(6)^6 = 1$

and so on.....

- Compare matrix inverse with 6 times the complex conjugate (element by element)

$$C_{012345}^{-1} - 6\cdot\overline{C_{012345}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Set tolerance value.....

Replace the 1/n term with SQRT(n)/n = 1/SQRT(n) to give a power invariant transform:

$$C_{alt} := \frac{1}{\sqrt{6}}\cdot\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a(6) & a(6)^2 & a(6)^3 & a(6)^4 & a(6)^5 \\ 1 & a(6)^2 & a(6)^4 & a(6)^6 & a(6)^8 & a(6)^{10} \\ 1 & a(6)^3 & a(6)^6 & a(6)^9 & a(6)^{12} & a(6)^{15} \\ 1 & a(6)^4 & a(6)^8 & a(6)^{12} & a(6)^{16} & a(6)^{20} \\ 1 & a(6)^5 & a(6)^{10} & a(6)^{15} & a(6)^{20} & a(6)^{25} \end{pmatrix}$$

$$C_{alt}^{-1} - \overline{C_{alt}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Finally, we can derive the reverse transformation:

We know that: $V_{b1} = a^{-1} \cdot V_{a1}$ This time we are trying to shift Va1 to Vb1

Note that:

$a^n = 1$ and $a^n \cdot a^{-1} = a^{n-1}$ Effectively added 360 degrees

Therefore: $a^{-1} = a^{n-1}$

- This repeats for integer multiples of n

Check with actual values:

$a(n)^{-1} - a(n)^{n-1} = 0$ $a(n)^{-2} - a(n)^{n-2} = 0$

"1" term:

$V_{b1} = a^{n-1} \cdot V_{a1}$

$V_{c1} = a^{n-2} \cdot V_{a1}$

$V_{d1} = a^{n-3} \cdot V_{a1}$

etc.

"2" term:

$V_{b2} = a^{n-2} \cdot V_{a2}$

$V_{c2} = a^{n-4} \cdot V_{a2}$

$V_{d2} = a^{n-6} \cdot V_{a2}$

etc.

"n-1" term:

$V_{b\_nm1} = a \cdot V_{a\_nm1}$

$V_{c\_nm1} = a^2 \cdot V_{a\_nm1}$

$V_{d\_nm1} = a^3 \cdot V_{a\_nm1}$

etc.

$n := 6$

$$A_{012345} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^{-1} & a^{-2} & a^{-3} & a^{-4} & a^{-(n-1)} \\ 1 & a^{-2} & a^{-4} & a^{-6} & a^{-8} & a^{-2(n-1)} \\ 1 & a^{-3} & a^{-6} & a^{-9} & a^{-12} & a^{-3(n-1)} \\ 1 & a^{-4} & a^{-8} & a^{-12} & a^{-16} & a^{-4(n-1)} \\ 1 & a^{-(n-1)} & a^{-2(n-1)} & a^{-3(n-1)} & a^{-4(n-1)} & a^{-5(n-1)} \end{bmatrix}$$

- Lets rewrite this a bit, realizing that n = 6

$$A_{012345} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^{n-1} & a^{n-2} & a^{n-3} & a^{n-4} & a^{n-5} \\ 1 & a^{n-2} & a^{n-4} & a^{2n-6} & a^{2n-8} & a^{2n-10} \\ 1 & a^{n-3} & a^{2n-6} & a^{2n-9} & a^{2n-12} & a^{3n-15} \\ 1 & a^{n-4} & a^{2n-8} & a^{2n-12} & a^{3n-16} & a^{4n-20} \\ 1 & a^{n-5} & a^{2n-10} & a^{3n-15} & a^{4n-20} & a^{4n-25} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^5 & a^4 & a^3 & a^2 & a \\ 1 & a^4 & a^2 & a^0 & a^4 & a^2 \\ 1 & a^3 & a^0 & a^3 & a^0 & a^3 \\ 1 & a^2 & a^4 & a^0 & a^2 & a^4 \\ 1 & a & a^2 & a^3 & a^4 & a^5 \end{pmatrix}$$

$$A_{012345} := \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a(n)^5 & a(n)^4 & a(n)^3 & a(n)^2 & a(n) \\ 1 & a(n)^4 & a(n)^2 & a(n)^0 & a(n)^4 & a(n)^2 \\ 1 & a(n)^3 & a(n)^0 & a(n)^3 & a(n)^0 & a(n)^3 \\ 1 & a(n)^2 & a(n)^4 & a(n)^0 & a(n)^2 & a(n)^4 \\ 1 & a(n) & a(n)^2 & a(n)^3 & a(n)^4 & a(n)^5 \end{pmatrix}$$

$$C_{012345}^{-1} - A_{012345} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$