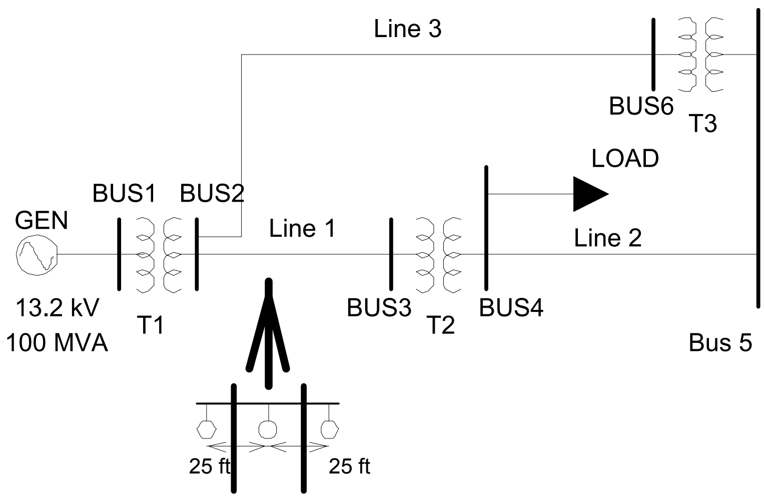
ECE 523
Per Unit Example

Define Units: $MVA := 1000kW$ $MW := MVA$ $MVAR := MW$ $pu := 1$

For the system below find: Given the system below do the following:



Where:

T1: Δ-Y, 13.8 kV:230kV, 200 MVA, X_{l1} = 10%

T2: Y-Δ, 220 kV:132kV, 100 MVA, X_{l2} = 10%

T3: Y-Δ, 220 kV:132kV, 500 MVA, X_{l3} = 10%

Line 2: 40 miles long, z' = 0.15 + j0.75 Ω/mile

Line 3: 190 miles long, z' = 0.15 + j0.75 Ω/mile and y'= j5.0 x 10^{-6}Mho/mile

Line 1: Tower as shown in figure. Conductor radius = 0.5 inches, with 3 conductors per bundle (18 in spacing). The line is 120 miles long and z'= j0.524 ohm/mi and y' = j7.962x10 $^{-6}$Mho/mile

A. Find the impedance of each line in Ohm and Admittance in Mhos

Line 1: $Length1 := 120mi$

$$Zline1 := \left(0 + j\cdot 0.524\frac{ohm}{mi}\right)\cdot Length1 \qquad Zline1 = 62.88i\,ohm$$

$$Yline1 := j\cdot 7.962\cdot 10^{-6}\frac{mho}{mi}\cdot Length1 \qquad Yline1 = 9.55i\times 10^{-4}\,mho$$

Line 2:

$$z2prime := (0.15 + j\cdot 0.75)\frac{ohm}{mi} \qquad length2 := 40mi$$

$$Zline2 := z2prime\cdot length2 \qquad Zline2 = (6 + 30i)\,ohm$$

Line 3: Use the long line model since it is over 150 miles long.

$$z3prime := (0.15 + j\cdot 0.75)\frac{ohm}{mi} \qquad length3 := 190mi$$

$$y3prime := j\cdot 5\cdot 10^{-6}\frac{mho}{mi}$$

$$Zline3 := z3prime\cdot length3 \qquad Zline3 = (28.5 + 142.5i)\,ohm$$

$$Yline3 := y3prime\cdot length3 \qquad Yline3 = 9.5i \times 10^{-4}\,mho$$

B Find the per unit equivalent circuit. Use a 100 MVA base, and the generator voltage as your reference voltage base.

$$Vbase1 := 13.2kV$$

$$Sbase := 100MVA$$

$$Vbase2 := Vbase1\cdot\left(\frac{230}{13.8}\right) \qquad Vbase2 = 220\,kV$$

$$Zbase2 := \frac{Vbase2^2}{Sbase} \qquad Zbase2 = 484\,ohm$$

$$Vbase3 := Vbase2\cdot\left(\frac{132}{220}\right) \qquad Vbase3 = 132\,kV$$

$$Zbase3 := \frac{Vbase3^2}{Sbase} \qquad Zbase3 = 174.24\,ohm$$

Per unit models for transformers (with change of base)

Transformer T1:

$$Xl1\_old := 0.1 \qquad Sb1old := 200MVA \qquad Vb1old := 13.8kV$$

$$X\_l1\_new := Xl1\_old \cdot \left(\frac{Vb1old}{Vbase1}\right)^2 \cdot \left(\frac{Sbase}{Sb1old}\right) \qquad X\_l1\_new = 0.05$$

Transformer T2:

$Xl2\_old := 0.1$ $\qquad Sb2old := 100MVA$

$Vb2old := 220kV$

$$X\_l2\_new := Xl2\_old \cdot \left(\frac{Vb2old}{Vbase2}\right)^2 \cdot \left(\frac{Sbase}{Sb2old}\right) \qquad X\_l2\_new = 0.1\,pu$$

Transformer T3:

$Xl3\_old := 0.1$ $\qquad Sb3old := 500MVA$

$Vb2old := 220kV$

$$X\_l3\_new := Xl3\_old \cdot \left(\frac{Vb1old}{Vbase1}\right)^2 \cdot \left(\frac{Sbase}{Sb3old}\right) \qquad X\_l3\_new = 0.02\,pu$$

Per unit models for the lines.

Line 1: Use Zbase for zone 2

$$Zline1pu := \frac{Zline1}{Zbase2} \qquad Zline1pu = 0.13i\,pu$$

Note that Ybase=1/Zbase so we can multiply by Zbase instead of dividing by Ybase.

$$Yline1pu := Yline1 \cdot Zbase2 \qquad Yline1pu = 0.46i\,pu \qquad \frac{Yline1pu}{2} = 0.23i\,pu$$

Line 2: Use Zbase for zone 3

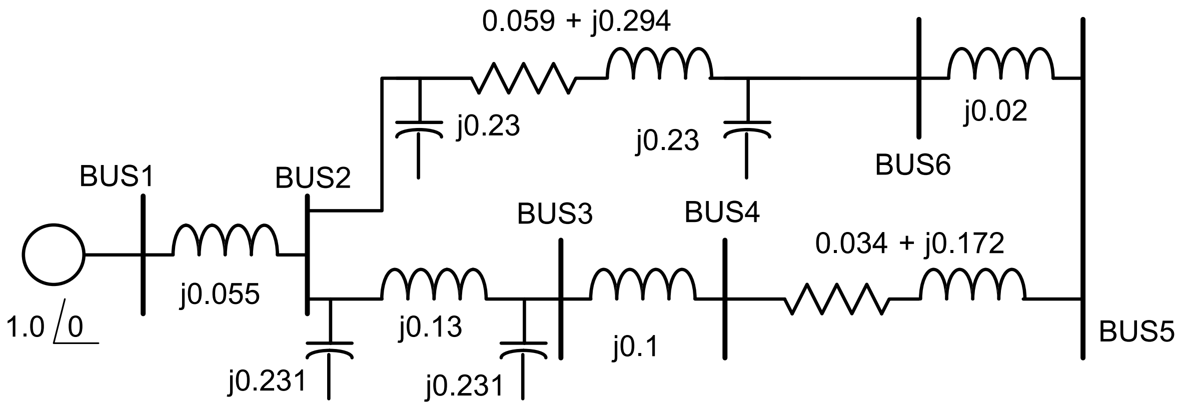
$$Zline2pu := \frac{Zline2}{Zbase3} \qquad Zline2pu = (0.03 + 0.17i)\,pu$$

Line 3: Use Zbase for zone 2

$$Zline3pu := \frac{Zline3}{Zbase2} \qquad Zline3pu = (0.06 + 0.29i)\,pu$$

$$Yline3pu := Yline3 \cdot Zbase2 \qquad Yline3pu = 0.46i\,pu \qquad \frac{Yline3pu}{2} = 0.23i\,pu$$

Per unit diagram:

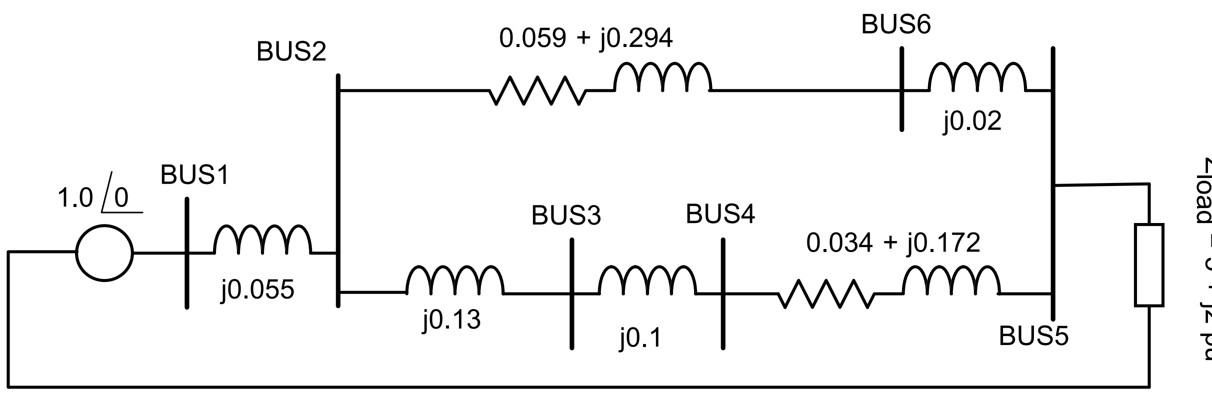


Now add a load at bus 5, and calculate the following assuming a source voltage of 1.0 at an angle of 0 degrees.

1) Calculate Vload in per unit and in kV
2) Calculate the source current, load current and the current in each transmission line in per unit and in Amps.

This comes down to circuit analysis. However, notice that the shunt capacitances will make this a very difficult problem to solve.

- For the moment, neglect the capacitances. We will discuss solving a circuit like this with multiple sources and multiple shunt elements when we start discussing matrix methods and power flow calculations later in the course.
- So we have the following equivalent circuit.



- Note that we have two parallel paths to reach bus 5 from the source. We will need to combine them and later use current divider equations to determine the line currents.

Net series impedance for line 3 and transformer T3:

$Ztop := Zline3pu + j \cdot X\_l3\_new$ $\quad Ztop = (0.06 + 0.32i)\,pu$

Net series impedance for line 1, transformer T2, and line 2:

$Zbottom := Zline1pu + jX\_l2\_new + Zline2pu$ $\quad Zbottom = (0.03 + 0.4i)\,pu$

The equivalent impedance (with parallel circuit) from the source to the load is:

$$Zeq := jX\_l1\_new + \left(\frac{1}{Zbottom} + \frac{1}{Ztop}\right)^{-1} \qquad Zeq = (0.03 + 0.23i)\,pu$$

Define load impedance and source voltage:

$Zload := 5pu + j \cdot 2pu$ $\quad Vs := 1pu \cdot e^{j \cdot 0deg}$

$arg(Zload) = 21.8\,deg$ $\quad pfload := \cos(arg(Zload))$ $\quad pfload = 0.93$ lagging

Load impedance in Ohms:

$Zload_{ohm} := Zload \cdot Zbase3$ $\quad \left|Zload_{ohm}\right| = 938.31\,\Omega$ fairly large

Calculate source current in per unit:

$$Isrcpu := \frac{Vs}{Zeq + Zload} \qquad |Isrcpu| = 0.18\,pu$$

$\theta_{isrc} := arg(Isrcpu)$ $\quad \theta_{isrc} = -23.95\,deg$

$Iloadpu := Isrcpu$ Only 1 load and 1 source and no other shunt branches.

$Vloadpu := Iloadpu \cdot Zload$ $\quad |Vloadpu| = 0.979\,pu$ or as a percentage:

$|Vloadpu| = 97.9\,\%$

$\theta_{vload} := arg(Vloadpu)$ $\quad \theta_{vload} = -2.15\,deg$

Voltage in kV:

Line to line voltage magnitude

$$magV_{ll\_load} := |Vloadpu| \cdot Vbase3 \qquad magV_{ll\_load} = 129.28\,kV$$

Line to neutral magnitude and angle:

$$V_{ln\_load} := \frac{magV_{ll\_load}}{\sqrt{3}} \cdot e^{j \cdot \theta_{vload}} \qquad \left|V_{ln\_load}\right| = 74.64\,kV$$

Associate per unit angle with V_{ln}

Calculate power factor from current and voltage:

$$PF_{load} := \cos\left(\theta_{vload} - \theta_{isrc}\right) \qquad PF_{load} = 0.93$$

lagging

Same as calculated from load impedance as one should expect.

As long as we're at it, lets calculate P and Q at the load:

$$Sloadpu := Vloadpu \cdot \overline{Iloadpu} \qquad Sloadpu = (0.17 + 0.07i)\,pu$$

Convert to MVA:

$$Sload_{MVA} := Sloadpu \cdot Sbase \qquad Sload_{MVA} = (16.54 + 6.62i)\,MVA$$

So this load does not draw much power compared to the 100 MVA base and the ratings of the equipment (i.e. a big impedance).

Now find the currents in Amperes. First we need the current bases for the three zones:

$$Ibase1 := \frac{Sbase}{\sqrt{3} \cdot Vbase1} \qquad Ibase1 = 4373.87\,A$$

$$Ibase2 := \frac{Sbase}{\sqrt{3} \cdot Vbase2} \qquad Ibase2 = 262.43\,A$$

$$Ibase3 := \frac{Sbase}{\sqrt{3} \cdot Vbase3} \qquad Ibase3 = 437.39\,A$$

Source current: $Isource_{Amps} := Isrcpu \cdot Ibase1$ $\quad \left|Isource_{Amps}\right| = 795.46\,A$

$\theta_{isrc} = -0.42\,pu$ unchanged if we use line to nuetral voltage angles.

Load current: $Iload_{Amps} := Iloadpu \cdot Ibase3$ $\quad \left|Iload_{Amps}\right| = 79.55\,A$

The angle of the load current matches that of the source current since the phase shifts of the two Y-Δ transformers cancel each other (i.e. it is what the text book calls a Normal System).

Line currents (now we need to use current dividers). The Y-Δ phase shifts are often ignored in per unit calculations unless we are looking at fault detection circuitry.

$$Itop := Isrcpu \cdot \left(\frac{Zbottom}{Ztop + Zbottom}\right) \qquad |Itop| = 0.1\,pu$$

$$\theta_{Itoppu} := \arg(Itop) \qquad \theta_{Itoppu} = -21.44\,deg$$

$$Ibottom := Isrcpu \cdot \left(\frac{Ztop}{Ztop + Zbottom}\right) \qquad |Ibottom| = 0.08\,pu$$

$$\theta_{Ibottompu} := \arg(Ibottom) \qquad \theta_{Ibottompu} = -27.1\,deg$$

As a check, since the magnitude of Itop is larger than the magnitude of Ibottom, we expect $|Ztop| < |Zbottom|$

$$|Ztop| = 0.32 \qquad |Zbottom| = 0.4$$

So the answer above is reasonable.

Now find the current in Amperes (we will put the Y-Δ shifts back in now).

Shift for T1: The Y side is the high voltage side, so we add 30 degrees to the equivalent voltage on the Δ side (always compare line-to-line with line-to-line or line-to-neutral with line-to-neutral). The current will have the same shift (as one would expect to maintain the same power factor).

Shift for T2 and T3: The Y side is the high voltage side for each, so we add 30 degrees to the equivalent voltage on the Δ side (always compare line-to-line with line-to-line or line-to-neutral with line-to-neutral). The current will have the same shift (as one would expect to maintain the same power factor). Since we are crossing in the opposite direction as we move to the load, this cancels the shift from T1.

$Iline1_{pu} := Ibottom$

$Iline1_{amps} := Iline1_{pu} \cdot Ibase2 \cdot e^{j \cdot 30deg}$ $\qquad \left| Iline1_{amps} \right| = 21.2\,A$

$\arg\left(Iline1_{amps}\right) = 2.9\,deg$

$Iline2_{pu} := Ibottom$

$Iline2_{amps} := Iline2_{pu} \cdot Ibase3 \cdot e^{j \cdot 0deg}$ $\qquad \left| Iline2_{amps} \right| = 35.33\,A$

No shift, since this is after transformer T2. $\qquad \arg\left(Iline2_{amps}\right) = -27.1\,deg$

Also notice that the magnitude is bigger than the current in line 1, after crossing the transformer.

$Iline3_{pu} := Itop$

$Iline3_{amps} := Iline3_{pu} \cdot Ibase2 \cdot e^{j \cdot 30deg}$ $\qquad \left| Iline3_{amps} \right| = 26.59\,A$

$\arg\left(Iline3_{amps}\right) = 8.56\,deg$

Finally, suppose we also want the current through one of the phases of the delta in Transformer T1. We can find this several ways.

1. Transform the phase currents in the Y connected winding.

$IwyeT1 := Isrcpu \cdot Ibase2 \cdot e^{j \cdot 30deg}$ $\qquad \left| IwyeT1 \right| = 47.73\,A$

$\arg(IwyeT1) = 6.05\,deg$

as a check:

$$IwyeT1 - Iline1_{amps} - Iline3_{amps} = 0\,A$$

$$IdeltaAB := IwyeT1 \cdot \frac{\frac{230}{\sqrt{3}}}{13.8}$$

Note that this is the turns ratio of the winding itself, not
the voltage transformation ratio.

$\left| IdeltaAB \right| = 459.26\,A$ $\qquad \arg(IdeltaAB) = 6.05\,deg$

similarly we have: $\quad IdeltaBC := IdeltaAB \cdot e^{-j \cdot 120deg} \qquad IdeltaCA := IdeltaAB \cdot e^{j \cdot 120deg}$

We expect to see $I_{line\_delta} := IdeltaAB - IdeltaCA$

$$I_{line\_delta} = (726.97 - 322.92i)\,A$$

which is the same as:

$$Isource_{Amps} = (726.97 - 322.92i)\,A$$

2. We can also just use the standard current relationships between the phase currents and line currents in a delta.

$$I_{delta\_source} := \left(\frac{Isource_{Amps}}{\sqrt{3}}\right)e^{j\cdot 30deg}$$

$$\left|I_{delta\_source}\right| = 459.26\,A$$

$$\arg\left(I_{delta\_source}\right) = 6.05\,deg$$

The angle change depends on the polarities of the currents in the delta, so the +30 degrees could be -30 degrees if the Delta is the higher voltage winding.