EE 523: Line Constants: Ground Wire and Bundled Conductors

A. Original Case

AC Resistance from table:

$Rac := 0.278\frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$\mu_0 := 4\cdot\pi\cdot 10^{-7}\frac{H}{m}$ $\rho := 100ohm\cdot m$ Conductor GMR from table:

$Ds := 0.01668ft$

$Zperlength := Rperlength + j\cdot 2\cdot\pi\cdot freq\cdot Lperlength$

$$Zperlength = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix}\frac{ohm}{mi}$$

If length = 40 miles: $Zline := Zperlength\cdot 40mi$

$$Zline = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix}\Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$Z012 := A_{012}^{-1} \cdot Zline \cdot A_{012}$$

$$Z012 = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

By comparsion:

$$Dm := (Dab \cdot Dbc \cdot Dac)^{\frac{1}{3}}$$

$$Z_1 := \left(Rac + j \cdot 2 \cdot \pi \cdot freq \cdot Indcons \cdot \ln\left(\frac{Dm}{Ds}\right)\right) \cdot 40mi$$

- ***Transposition***

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Measure of imbalance

define unit: $mS := \frac{S}{1000}$

Original System

$$Y012 := Z012^{-1}$$

$$Y012 = \begin{pmatrix} 1.7687 - 8.6713i & 0.3107 + 0.0104i & -0.1464 - 0.2743i \\ -0.1464 - 0.2743i & 9.7166 - 27.8455i & -1.9327 - 0.2845i \\ 0.3107 + 0.0104i & 0.72 + 1.816i & 9.7166 - 27.8455i \end{pmatrix} mS$$

First transposed case:

$$Y0121 := Z0121^{-1}$$

$$Y0121 = \begin{pmatrix} 1.7647 - 8.6671i & -0.0572 - 0.0523i & 0.0739 + 0.0256i \\ 0.0739 + 0.0256i & 9.6082 - 27.7751i & 0.3356 + 0.3876i \\ -0.0572 - 0.0523i & -0.4562 - 0.2345i & 9.6082 - 27.7751i \end{pmatrix} mS$$

Second transposed case:

$$Y0123 := Z0123^{-1}$$

$$Y0123 = \begin{pmatrix} 1.7655 - 8.668i & 0.0235 + 0.1483i & -0.1436 + 0.0535i \\ -0.1436 + 0.0535i & 9.6322 - 27.7908i & -0.0464 - 1.0228i \\ 0.0235 + 0.1483i & 1.0058 - 0.1965i & 9.6322 - 27.7908i \end{pmatrix} mS$$

Fully Transposed Case

$$Y0126 := Z0126^{-1}$$

$$Y0126 = \begin{pmatrix} 1.7645 - 8.6668i & 0 & 0 \\ 0 & 9.6002 - 27.7698i & 0 \\ 0 & 0 & 9.6002 - 27.7698i \end{pmatrix} mS$$

Imbalance factors:

$$M_{0\_original} := \frac{Y012_{0,1}}{Y012_{1,1}} \qquad \left|M_{0\_original}\right| = 0.0105$$

$$M_{2\_original} := \frac{Y012_{2,1}}{Y012_{1,1}} \qquad \left|M_{2\_original}\right| = 0.0662$$

$$M_{0\_trans1} := \frac{Y0121_{0,1}}{Y0121_{1,1}} \qquad \left|M_{0\_trans1}\right| = 2.6354 \times 10^{-3}$$

$$M_{2\_trans1} := \frac{Y0121_{2,1}}{Y0121_{1,1}} \qquad \left|M_{2\_trans1}\right| = 0.0175$$

$$M_{0\_trans2} := \frac{Y0123_{0,1}}{Y0123_{1,1}} \qquad \left|M_{0\_trans2}\right| = 5.1064 \times 10^{-3}$$

$$M_{2\_trans2} := \frac{Y0123_{2,1}}{Y0123_{1,1}} \qquad \left|M_{2\_trans2}\right| = 0.0348$$

$$M_{0\_trans3} := \frac{Y0126_{0,1}}{Y0126_{1,1}} \qquad \left|M_{0\_trans3}\right| = 0$$

$$M_{2\_trans3} := \frac{Y0126_{2,1}}{Y0126_{1,1}} \qquad \left|M_{2\_trans3}\right| = 0$$

B. Example with Ground Wire Added

AC Resistance from table:

$$Rgw := 4.0\frac{ohm}{mi} \qquad Rselfgw := Rgw + Rd$$

$$Rperlength := \begin{pmatrix} Rself & Rd & Rd & Rd \\ Rd & Rself & Rd & Rd \\ Rd & Rd & Rself & Rd \\ Rd & Rd & Rd & Rselfgw \end{pmatrix}$$

$$Rperlength = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 4.0953 \end{pmatrix} \frac{ohm}{mi}$$

Calculate GMR from conductor diameter

$$dia := 0.528in$$

$$GMR := e^{\frac{-1}{4}} \cdot \frac{dia}{2} \qquad GMR = 0.01713\,ft$$

$$Dsgw := 1\cdot 10^{-3}ft$$

$$Dagw := \sqrt{(10ft)^2 + (15ft)^2} \qquad Dcgw := Dagw \qquad Dbgw := 15ft$$

$$Lperlength := Indcons\cdot\begin{pmatrix} \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Dab}\right) & \ln\left(\frac{De}{Dac}\right) & \ln\left(\frac{De}{Dagw}\right) \\ \ln\left(\frac{De}{Dab}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Dbc}\right) & \ln\left(\frac{De}{Dbgw}\right) \\ \ln\left(\frac{De}{Dac}\right) & \ln\left(\frac{De}{Dbc}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Dcgw}\right) \\ \ln\left(\frac{De}{Dagw}\right) & \ln\left(\frac{De}{Dbgw}\right) & \ln\left(\frac{De}{Dcgw}\right) & \ln\left(\frac{De}{Dsgw}\right) \end{pmatrix}$$

$$Lperlength = \begin{pmatrix} 3.8711 & 1.8123 & 1.5892 & 1.6227 \\ 1.8123 & 3.8711 & 1.8123 & 1.6818 \\ 1.5892 & 1.8123 & 3.8711 & 1.6227 \\ 1.6227 & 1.6818 & 1.6227 & 4.7769 \end{pmatrix}\frac{mH}{mi}$$

$$Zperlength := Rperlength + j\cdot 2\cdot\pi\cdot freq\cdot Lperlength$$

$$Zperlength = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i & 0.095 + 0.612i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.634i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.612i \\ 0.095 + 0.612i & 0.095 + 0.634i & 0.095 + 0.612i & 4.095 + 1.801i \end{pmatrix}\frac{ohm}{mi}$$

If length = 40 miles: $\text{Zline} := \text{Zperlength}\cdot 40\text{mi}$

$$\text{Zline} = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i & 3.812 + 24.469i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 25.362i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 24.469i \\ 3.812 + 24.469i & 3.812 + 25.362i & 3.812 + 24.469i & 163.812 + 72.033i \end{pmatrix} \Omega$$

Reduce to equivalent 3x3 matrix by removing 4th row and column

$\text{Za} := \text{submatrix}(\text{Zline}, 0, 2, 0, 2)$ $\text{Zb} := \text{submatrix}(\text{Zline}, 0, 2, 3, 3)$

$\text{Zc} := \text{submatrix}(\text{Zline}, 3, 3, 0, 2)$ $\text{Zd} := \text{submatrix}(\text{Zline}, 3, 3, 3, 3)$

$$\text{Zabceq} := \text{Za} - \text{Zb}\cdot\text{Zd}^{-1}\cdot\text{Zc}$$

$$\text{Zabceq} = \begin{pmatrix} 17.5007 + 56.1058i & 6.4847 + 24.9946i & 6.3807 + 21.6968i \\ 6.4847 + 24.9946i & 17.7129 + 55.971i & 6.4847 + 24.9946i \\ 6.3807 + 21.6968i & 6.4847 + 24.9946i & 17.5007 + 56.1058i \end{pmatrix} \Omega$$

$$\text{Z012}_{\text{GW}} := \text{A}_{012}^{-1}\cdot\text{Zabceq}\cdot\text{A}_{012}$$

$$\text{Z012}_{\text{GW}} = \begin{pmatrix} 30.4715 + 103.8515i & 0.8604 - 0.6184i & -0.9658 - 0.4359i \\ -0.9658 - 0.4359i & 11.1214 + 32.1655i & -1.9436 + 1.1206i \\ 0.8604 - 0.6184i & 1.9422 + 1.1229i & 11.1214 + 32.1655i \end{pmatrix} \Omega$$

For comparison, without the groundwire:

$$Z012old := {A_{012}}^{-1} \cdot Za \cdot A_{012}$$

$$Z012old = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

As a check:

$$Z012 - Z012old = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Omega$$

Unbalance Factors:

$$Y012_{GW} := {Z012_{GW}}^{-1}$$

$$M0_{gw} := \frac{{Y012_{GW}}_{0,1}}{{Y012_{GW}}_{1,1}} \qquad \left|M0_{gw}\right| = 0.0104$$

Compare to case without GW:

$$\left|M_{0\_original}\right| = 0.0105 \qquad \text{Small change}$$

$$\left| \frac{\left|M_{0\_original}\right| - \left|M0_{gw}\right|}{\left|M_{0\_original}\right|} \right| = 1.2967\,\%$$

$$M2_{gw} := \frac{Y012_{GW_{2,1}}}{Y012_{GW_{1,1}}} \qquad \left|M2_{gw}\right| = 0.0662$$

Compare to case without GW:

$$\left|M_{2\_original}\right| = 0.0662$$ No change

$$\left| \frac{\left|M_{2\_original}\right| - \left|M2_{gw}\right|}{\left|M_{2\_original}\right|} \right| = 1.5002 \times 10^{-3}\,\%$$

C. Example with Two Conductor Bundles (no ground wire)

AC Resistance from table

$Rac_{bund} := 0.117\frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$Rself_{bund} := Rac_{bund} + Rd$ $Rself_{bund} = 0.2123\frac{ohm}{mi}$

$$R' := \begin{pmatrix} Rself_{bund} & Rd & Rd & Rd & Rd & Rd \\ Rd & Rself_{bund} & Rd & Rd & Rd & Rd \\ Rd & Rd & Rself_{bund} & Rd & Rd & Rd \\ Rd & Rd & Rd & Rself_{bund} & Rd & Rd \\ Rd & Rd & Rd & Rd & Rself_{bund} & Rd \\ Rd & Rd & Rd & Rd & Rd & Rself_{bund} \end{pmatrix}$$

Conductor GMR from table:

$Ds := 0.0375ft$

Spacing:

Within the bundle:

$Da1a2 := 1.5ft$ $Db1b2 := 1.5ft$ $Dc1c2 := 1.5ft$

Between Phases

$Da1b1 := 24ft$ $Da1b2 := Da1b1 + Db1b2$ $Da1b2 = 25.5\,ft$

$Da2b1 := Da1b1 - Da1a2$ $\quad Da2b1 = 22.5\,ft$ $\quad Da2b2 := 24ft$

$Da1c1 := 48ft$ $\quad Da1c2 := Da1c1 + Dc1c2$ $\quad Da1c2 = 49.5\,ft$

$Da2c1 := Da1c1 - Da1a2$ $\quad Da2c1 = 46.5\,ft$ $\quad Da2c2 := 48ft$

$Db1c1 := 24ft$ $\quad Db1c2 := Db1c1 + Dc1c2$ $\quad Db1c2 = 25.5\,ft$

$Db2c1 := Db1c1 - Db1b2$ $\quad Db2c1 = 22.5\,ft$ $\quad Db2c2 := 24ft$

$$L' := Indcons\begin{pmatrix} \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Da1c2}\right) \\ \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Da2b1}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db1c2}\right) \\ \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2c1}\right) & \ln\left(\frac{De}{Db2c1}\right) & \ln\left(\frac{De}{Dc1c2}\right) \\ \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da2b1}\right) & \ln\left(\frac{De}{Da2c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Da2c2}\right) \\ \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db2c1}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db2c2}\right) \\ \ln\left(\frac{De}{Da1c2}\right) & \ln\left(\frac{De}{Db1c2}\right) & \ln\left(\frac{De}{Dc1c2}\right) & \ln\left(\frac{De}{Da2c2}\right) & \ln\left(\frac{De}{Db2c2}\right) & \ln\left(\frac{De}{Ds}\right) \end{pmatrix}$$

$$Z' := R' + j \cdot 2 \cdot \pi \cdot freq \cdot L'$$

$$Z' = \begin{pmatrix} 0.212+1.361i & 0.095+0.577i & 0.095+0.493i & 0.095+0.913i & 0.095+0.57i & 0.095+0.489i \\ 0.095+0.577i & 0.212+1.361i & 0.095+0.577i & 0.095+0.585i & 0.095+0.913i & 0.095+0.57i \\ 0.095+0.493i & 0.095+0.577i & 0.212+1.361i & 0.095+0.497i & 0.095+0.585i & 0.095+0.913i \\ 0.095+0.913i & 0.095+0.585i & 0.095+0.497i & 0.212+1.361i & 0.095+0.577i & 0.095+0.493i \\ 0.095+0.57i & 0.095+0.913i & 0.095+0.585i & 0.095+0.577i & 0.212+1.361i & 0.095+0.577i \\ 0.095+0.489i & 0.095+0.57i & 0.095+0.913i & 0.095+0.493i & 0.095+0.577i & 0.212+1.361i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

If length = 40 miles: $\text{Zline} := Z' \cdot 40\text{mi}$

$$\text{Zline} = \begin{pmatrix} 8.492+54.442i & 3.812+23.08i & 3.812+19.716i & 3.812+36.537i & 3.812+22.786i & 3.812+19.567i \\ 3.812+23.08i & 8.492+54.442i & 3.812+23.08i & 3.812+23.394i & 3.812+36.537i & 3.812+22.786i \\ 3.812+19.716i & 3.812+23.08i & 8.492+54.442i & 3.812+19.87i & 3.812+23.394i & 3.812+36.537i \\ 3.812+36.537i & 3.812+23.394i & 3.812+19.87i & 8.492+54.442i & 3.812+23.08i & 3.812+19.716i \\ 3.812+22.786i & 3.812+36.537i & 3.812+23.394i & 3.812+23.08i & 8.492+54.442i & 3.812+23.08i \\ 3.812+19.567i & 3.812+22.786i & 3.812+36.537i & 3.812+19.716i & 3.812+23.08i & 8.492+54.442i \end{pmatrix} \Omega$$

$\text{Za} := \text{submatrix}(Z', 0, 2, 0, 2)$

$$\text{Za} = \begin{pmatrix} 0.2123+1.3611i & 0.0953+0.577i & 0.0953+0.4929i \\ 0.0953+0.577i & 0.2123+1.3611i & 0.0953+0.577i \\ 0.0953+0.4929i & 0.0953+0.577i & 0.2123+1.3611i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$Zb := \text{submatrix}(Z', 0, 2, 3, 5)$

$$Zb = \begin{pmatrix} 0.0953 + 0.9134i & 0.0953 + 0.5697i & 0.0953 + 0.4892i \\ 0.0953 + 0.5848i & 0.0953 + 0.9134i & 0.0953 + 0.5697i \\ 0.0953 + 0.4968i & 0.0953 + 0.5848i & 0.0953 + 0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$Zc := \text{submatrix}(Z', 3, 5, 0, 2)$

$$Zc = \begin{pmatrix} 0.0953 + 0.9134i & 0.0953 + 0.5848i & 0.0953 + 0.4968i \\ 0.0953 + 0.5697i & 0.0953 + 0.9134i & 0.0953 + 0.5848i \\ 0.0953 + 0.4892i & 0.0953 + 0.5697i & 0.0953 + 0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$Zd := \text{submatrix}(Z', 3, 5, 3, 5)$

$$Zd = \begin{pmatrix} 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i \end{pmatrix} \frac{ohm}{mi}$$

- Modify matrix by performing: Va'b'c'-Vabc and Iabc + Ia'b'c'

$Zbnew := Zb - Za$

$Zcnew := Zc - Za$

$Zdnew := Za - Zb - Zc + Zd$

$$Zdnew = \begin{pmatrix} 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -1.1856i \times 10^{-4} \\ -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -4.7492i \times 10^{-4} \\ -1.1856i \times 10^{-4} & -4.7492i \times 10^{-4} & 0.234 + 0.8952i \end{pmatrix} \frac{ohm}{mi}$$

$$Zequiv := Za - Zbnew \cdot Zdnew^{-1} \cdot Zcnew$$

$$Zequiv = \begin{pmatrix} 0.1538 + 1.1372i & 0.0953 + 0.5771i & 0.0953 + 0.493i \\ 0.0953 + 0.5771i & 0.1538 + 1.1371i & 0.0953 + 0.5771i \\ 0.0953 + 0.493i & 0.0953 + 0.5771i & 0.1538 + 1.1372i \end{pmatrix} \frac{ohm}{mi}$$

$$Z_{line\_bund} := Zequiv \cdot 40mi$$

$$Z_{line\_bund} = \begin{pmatrix} 6.1526 + 45.4868i & 3.8121 + 23.0838i & 3.8112 + 19.7196i \\ 3.8121 + 23.0838i & 6.1531 + 45.485i & 3.8121 + 23.0838i \\ 3.8112 + 19.7196i & 3.8121 + 23.0838i & 6.1526 + 45.4868i \end{pmatrix} \Omega$$

$$Z012_{bund} := A_{012}{}^{-1} \cdot Z_{line\_bund} \cdot A_{012}$$

$$Z012_{bund} = \begin{pmatrix} 13.7764 + 89.411i & 0.9704 - 0.5608i & -0.9709 - 0.56i \\ -0.9709 - 0.56i & 2.3409 + 23.5238i & -1.9426 + 1.1221i \\ 0.9704 - 0.5608i & 1.9431 + 1.1213i & 2.3409 + 23.5238i \end{pmatrix} ohm$$

Compare to:

$$Z012 := \begin{pmatrix} 22.556 + j \cdot 110.80 & 0.971 - j \cdot 0.561 & -0.971 - j \cdot 0.561 \\ -0.971 - j \cdot 0.561 & 11.120 + j \cdot 32.165 & -1.942 + j \cdot 1.121 \\ 0.971 - j \cdot 0.561 & -1.942 + j \cdot 1.121 & 11.120 + j \cdot 32.165 \end{pmatrix} ohm$$

- Note that the off diagonal terms are almost identical, but the diagonal terms are smaller, due to the bundling.
- Compare positive sequence resistance to ac resistance of conductors.

$$Rac_{bund} \cdot 40mi = 4.68\,\Omega$$

$$Re\left(Z012_{bund_{1,1}}\right) = 2.3409\,\Omega$$

Versus: $\frac{Rac_{bund}}{2} \cdot 40mi = 2.34\,\Omega$

- Recall the conductors are in parallel on each phase....

D. Example with Three Conductor Bundles (no ground wire)

AC Resistance from table in appendix

$Rac_{bund3} := 0.117 \frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$Rself_{b3} := Rac_{bund3} + Rd$ $\qquad Rself_{b3} = 0.2123 \frac{ohm}{mi}$

$$R' := \begin{pmatrix} Rself_{b3} & Rd & Rd & Rd & Rd & Rd & Rd & Rd & Rd \\ Rd & Rself_{b3} & Rd & Rd & Rd & Rd & Rd & Rd & Rd \\ Rd & Rd & Rself_{b3} & Rd & Rd & Rd & Rd & Rd & Rd \\ Rd & Rd & Rd & Rself_{b3} & Rd & Rd & Rd & Rd & Rd \\ Rd & Rd & Rd & Rd & Rself_{b3} & Rd & Rd & Rd & Rd \\ Rd & Rd & Rd & Rd & Rd & Rself_{b3} & Rd & Rd & Rd \\ Rd & Rd & Rd & Rd & Rd & Rd & Rself_{b3} & Rd & Rd \\ Rd & Rd & Rd & Rd & Rd & Rd & Rd & Rself_{b3} & Rd \\ Rd & Rd & Rd & Rd & Rd & Rd & Rd & Rd & Rself_{b3} \end{pmatrix}$$

Conductor GMR from table:

$Ds := 0.0375ft$

Spacing:

Within the bundle:

$Da1a2 := 1.5ft$ $Db1b2 := 1.5ft$ $Dc1c2 := 1.5ft$

$Da1a3 := 1.5ft$ $Db1b3 := 1.5ft$ $Dc1c3 := 1.5ft$

$Da2a3 := 1.5ft$ $Db2b3 := 1.5ft$ $Dc2c3 := 1.5ft$

Between Phases $vert := \sqrt{18^2 - 9^2}\,in$ $vert = 1.299\,ft$

$Da1b1 := 24ft$ $\quad Da1b2 := Da1b1 + Db1b2$ $\quad Da1b2 = 25.5\,ft$

$Da1b3 := \sqrt{24.75^2 + 1.299^2}\,ft$ $\quad Da1b3 = 24.7841\,ft$

$Da2b1 := Da1b1 - Da1a2$ $\quad Da2b1 = 22.5\,ft$ $\quad Da2b2 := 24ft$

$Da2b3 := \sqrt{23.25^2 + 1.299^2}\,ft$ $\quad Da2b3 = 23.2863\,ft$

$Da3b1 := Da2b3$ $\quad Da3b2 := Da1b3$ $\quad Da3b3 := 24ft$

$Da1c1 := 48ft$ $\quad Da1c2 := Da1c1 + Dc1c2$ $\quad Da1c2 = 49.5\,ft$

$Da1c3 := \sqrt{48.75^2 + 1.299^2}\,ft$ $\quad Da1c3 = 48.7673\,ft$

$Da2c1 := Da1c1 - Da1a2$ $\quad Da2c1 = 46.5\,ft$ $\quad Da2c2 := 48ft$

$Da2c3 := \sqrt{47.25^2 + 1.299^2}\,ft$ $\quad Da2c3 = 47.2679\,ft$

$Da3c1 := Da2c3$ $\quad Da3c2 := Da1c3$ $\quad Da3c3 := 48ft$

$Db1c1 := 24ft$ $\quad Db1c2 := Db1c1 + Dc1c2$ $\quad Db1c2 = 25.5\,ft$

$Db1c3 := Da1b3$

$Db2c1 := Db1c1 - Db1b2$ $\quad Db2c1 = 22.5\,ft$ $\quad Db2c2 := 24ft$

$Db2c3 := Da2b3$

$Db3c1 := Db2c3 \quad Db3c2 := Db1c3$

$Db3c3 := 24ft$

$$L' := \frac{\mu_0}{2\pi}\cdot\begin{pmatrix}
\ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Da1c2}\right) & \ln\left(\frac{De}{Da1a3}\right) & \ln\left(\frac{De}{Da1b3}\right) & \ln\left(\frac{De}{Da1c3}\right) \\
\ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Da2b1}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db1c2}\right) & \ln\left(\frac{De}{Da3b1}\right) & \ln\left(\frac{De}{Db1b3}\right) & \ln\left(\frac{De}{Db1c3}\right) \\
\ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2c1}\right) & \ln\left(\frac{De}{Db2c1}\right) & \ln\left(\frac{De}{Dc1c2}\right) & \ln\left(\frac{De}{Da3c1}\right) & \ln\left(\frac{De}{Db3c1}\right) & \ln\left(\frac{De}{Dc1c3}\right) \\
\ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da2b1}\right) & \ln\left(\frac{De}{Da2c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Da2c2}\right) & \ln\left(\frac{De}{Da2a3}\right) & \ln\left(\frac{De}{Da2b3}\right) & \ln\left(\frac{De}{Da2c3}\right) \\
\ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db2c1}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db2c2}\right) & \ln\left(\frac{De}{Da3b2}\right) & \ln\left(\frac{De}{Db2b3}\right) & \ln\left(\frac{De}{Db2c3}\right) \\
\ln\left(\frac{De}{Da1c2}\right) & \ln\left(\frac{De}{Db1c2}\right) & \ln\left(\frac{De}{Dc1c2}\right) & \ln\left(\frac{De}{Da2c2}\right) & \ln\left(\frac{De}{Db2c2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da3c2}\right) & \ln\left(\frac{De}{Db3c2}\right) & \ln\left(\frac{De}{Dc2c3}\right) \\
\ln\left(\frac{De}{Da1a3}\right) & \ln\left(\frac{De}{Da3b1}\right) & \ln\left(\frac{De}{Da3c1}\right) & \ln\left(\frac{De}{Da2a3}\right) & \ln\left(\frac{De}{Da3b2}\right) & \ln\left(\frac{De}{Da3c2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da3b3}\right) & \ln\left(\frac{De}{Da3c3}\right) \\
\ln\left(\frac{De}{Da1b3}\right) & \ln\left(\frac{De}{Db1b3}\right) & \ln\left(\frac{De}{Db3c1}\right) & \ln\left(\frac{De}{Da2b3}\right) & \ln\left(\frac{De}{Db2b3}\right) & \ln\left(\frac{De}{Db3c2}\right) & \ln\left(\frac{De}{Da3b3}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db3c3}\right) \\
\ln\left(\frac{De}{Da1c3}\right) & \ln\left(\frac{De}{Db1c3}\right) & \ln\left(\frac{De}{Dc1c3}\right) & \ln\left(\frac{De}{Da2c3}\right) & \ln\left(\frac{De}{Db2c3}\right) & \ln\left(\frac{De}{Dc2c3}\right) & \ln\left(\frac{De}{Da3c3}\right) & \ln\left(\frac{De}{Db3c3}\right) & \ln\left(\frac{De}{Ds}\right)
\end{pmatrix}$$

$Z' := R' + j\cdot 2\cdot\pi\cdot freq\cdot L'$

$$Z' = \begin{pmatrix} 0.21+1.36i & 0.1+0.58i & 0.1+0.49i & 0.1+0.91i & 0.1+0.57i & 0.1+0.49i & 0.1+0.91i & 0.1+0.57i & 0.1+0.49i \\ 0.1+0.58i & 0.21+1.36i & 0.1+0.58i & 0.1+0.58i & 0.1+0.91i & 0.1+0.57i & 0.1+0.58i & 0.1+0.91i & 0.1+0.57i \\ 0.1+0.49i & 0.1+0.58i & 0.21+1.36i & 0.1+0.5i & 0.1+0.58i & 0.1+0.91i & 0.1+0.49i & 0.1+0.58i & 0.1+0.91i \\ 0.1+0.91i & 0.1+0.58i & 0.1+0.5i & 0.21+1.36i & 0.1+0.58i & 0.1+0.49i & 0.1+0.91i & 0.1+0.58i & 0.1+0.49i \\ 0.1+0.57i & 0.1+0.91i & 0.1+0.58i & 0.1+0.58i & 0.21+1.36i & 0.1+0.58i & 0.1+0.57i & 0.1+0.91i & 0.1+0.58i \\ 0.1+0.49i & 0.1+0.57i & 0.1+0.91i & 0.1+0.49i & 0.1+0.58i & 0.21+1.36i & 0.1+0.49i & 0.1+0.57i & 0.1+0.91i \\ 0.1+0.91i & 0.1+0.58i & 0.1+0.49i & 0.1+0.91i & 0.1+0.57i & 0.1+0.49i & 0.21+1.36i & 0.1+0.58i & 0.1+0.49i \\ 0.1+0.57i & 0.1+0.91i & 0.1+0.58i & 0.1+0.58i & 0.1+0.91i & 0.1+0.57i & 0.1+0.58i & 0.21+1.36i & 0.1+0.58i \\ 0.1+0.49i & 0.1+0.57i & 0.1+0.91i & 0.1+0.49i & 0.1+0.58i & 0.1+0.91i & 0.1+0.49i & 0.1+0.58i & 0.21+1.36i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$Z_L := Z' \cdot 40\text{mi}$$

$$Z_L = \begin{pmatrix} 8.49+54.44i & 3.81+23.08i & 3.81+19.72i & 3.81+36.54i & 3.81+22.79i & 3.81+19.57i & 3.81+36.54i & 3.81+22.92i & 3.81+19.64i \\ 3.81+23.08i & 8.49+54.44i & 3.81+23.08i & 3.81+23.39i & 3.81+36.54i & 3.81+22.79i & 3.81+23.23i & 3.81+36.54i & 3.81+22.92i \\ 3.81+19.72i & 3.81+23.08i & 8.49+54.44i & 3.81+19.87i & 3.81+23.39i & 3.81+36.54i & 3.81+19.79i & 3.81+23.23i & 3.81+36.54i \\ 3.81+36.54i & 3.81+23.39i & 3.81+19.87i & 8.49+54.44i & 3.81+23.08i & 3.81+19.72i & 3.81+36.54i & 3.81+23.23i & 3.81+19.79i \\ 3.81+22.79i & 3.81+36.54i & 3.81+23.39i & 3.81+23.08i & 8.49+54.44i & 3.81+23.08i & 3.81+22.92i & 3.81+36.54i & 3.81+23.23i \\ 3.81+19.57i & 3.81+22.79i & 3.81+36.54i & 3.81+19.72i & 3.81+23.08i & 8.49+54.44i & 3.81+19.64i & 3.81+22.92i & 3.81+36.54i \\ 3.81+36.54i & 3.81+23.23i & 3.81+19.79i & 3.81+36.54i & 3.81+22.92i & 3.81+19.64i & 8.49+54.44i & 3.81+23.08i & 3.81+19.72i \\ 3.81+22.92i & 3.81+36.54i & 3.81+23.23i & 3.81+23.23i & 3.81+36.54i & 3.81+22.92i & 3.81+23.08i & 8.49+54.44i & 3.81+23.08i \\ 3.81+19.64i & 3.81+22.92i & 3.81+36.54i & 3.81+19.79i & 3.81+23.23i & 3.81+36.54i & 3.81+19.72i & 3.81+23.08i & 8.49+54.44i \end{pmatrix} \Omega$$

$Za := \text{submatrix}(Z',0,2,0,2)$

$$Za = \begin{pmatrix} 0.2123+1.3611i & 0.0953+0.577i & 0.0953+0.4929i \\ 0.0953+0.577i & 0.2123+1.3611i & 0.0953+0.577i \\ 0.0953+0.4929i & 0.0953+0.577i & 0.2123+1.3611i \end{pmatrix} \frac{ohm}{mi}$$

$Zb1 := \text{submatrix}(Z',0,2,3,5)$

$$Zb1 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5697i & 0.0953+0.4892i \\ 0.0953+0.5848i & 0.0953+0.9134i & 0.0953+0.5697i \\ 0.0953+0.4968i & 0.0953+0.5848i & 0.0953+0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$Zb2 := \text{submatrix}(Z',0,2,6,8)$

$$Zb2 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5731i & 0.0953+0.491i \\ 0.0953+0.5807i & 0.0953+0.9134i & 0.0953+0.5731i \\ 0.0953+0.4948i & 0.0953+0.5807i & 0.0953+0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$Zc1 := \text{submatrix}(Z',3,5,0,2)$

$$Zc1 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5848i & 0.0953+0.4968i \\ 0.0953+0.5697i & 0.0953+0.9134i & 0.0953+0.5848i \\ 0.0953+0.4892i & 0.0953+0.5697i & 0.0953+0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$Zc2 := \text{submatrix}(Z',6,8,0,2)$

$$Zc2 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5807i & 0.0953+0.4948i \\ 0.0953+0.5731i & 0.0953+0.9134i & 0.0953+0.5807i \\ 0.0953+0.491i & 0.0953+0.5731i & 0.0953+0.9134i \end{pmatrix} \frac{ohm}{mi}$$

$$Zd := \text{submatrix}(Z', 3, 8, 3, 8)$$

$$Zd = \begin{pmatrix} 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i & 0.0953 + 0.9134i & 0.0953 + 0.5807i & 0.0953 + 0.4948i \\ 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.5731i & 0.0953 + 0.9134i & 0.0953 + 0.5807i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.491i & 0.0953 + 0.5731i & 0.0953 + 0.9134i \\ 0.0953 + 0.9134i & 0.0953 + 0.5731i & 0.0953 + 0.491i & 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.5807i & 0.0953 + 0.9134i & 0.0953 + 0.5731i & 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i \\ 0.0953 + 0.4948i & 0.0953 + 0.5807i & 0.0953 + 0.9134i & 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$Zbnew := \text{augment}(Zb1 - Za, Zb2 - Za)$$

$$Zcnew := \text{stack}(Zc1 - Za, Zc2 - Za)$$

$$Zbnew^T - Zcnew = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$Zcnew = \begin{pmatrix} -0.117 - 0.4476i & 7.8312i \times 10^{-3} & 3.8524i \times 10^{-3} \\ -7.3563i \times 10^{-3} & -0.117 - 0.4476i & 7.8312i \times 10^{-3} \\ -3.7339i \times 10^{-3} & -7.3563i \times 10^{-3} & -0.117 - 0.4476i \\ -0.117 - 0.4476i & 3.6633i \times 10^{-3} & 1.8651i \times 10^{-3} \\ -3.9008i \times 10^{-3} & -0.117 - 0.4476i & 3.6633i \times 10^{-3} \\ -1.9244i \times 10^{-3} & -3.9008i \times 10^{-3} & -0.117 - 0.4476i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$Zd1 := Za - Zb1 - Zc1 + \text{submatrix}(Zd, 0, 2, 0, 2)$$

$$Zd2 := Za - Zb2 - Zc1 + \text{submatrix}(Zd, 0, 2, 3, 5)$$

$Zd2 := Za - Zb2 - Zc1 + \text{submatrix}(Zd, 0, 2, 3, 5)$

$Zd3 := Za - Zb1 - Zc2 + \text{submatrix}(Zd, 3, 5, 0, 2)$

$Zd4 := Za - Zb2 - Zc2 + \text{submatrix}(Zd, 3, 5, 3, 5)$

$ZdA := \text{augment}(Zd1, Zd2)$ $\qquad$ $ZdB := \text{augment}(Zd3, Zd4)$

$$ZdA = \begin{pmatrix} 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -1.1856i \times 10^{-4} & 0.117 + 0.4476i & -2.6708i \times 10^{-4} & -6.2981i \times 10^{-5} \\ -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -2.0784i \times 10^{-4} & 0.117 + 0.4476i & -2.6708i \times 10^{-4} \\ -1.1856i \times 10^{-4} & -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -5.5575i \times 10^{-5} & -2.0784i \times 10^{-4} & 0.117 + 0.4476i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$ZdB = \begin{pmatrix} 0.117 + 0.4476i & -2.0784i \times 10^{-4} & -5.5575i \times 10^{-5} & 0.234 + 0.8952i & 2.3744i \times 10^{-4} & 5.9273i \times 10^{-5} \\ -2.6708i \times 10^{-4} & 0.117 + 0.4476i & -2.0784i \times 10^{-4} & 2.3744i \times 10^{-4} & 0.234 + 0.8952i & 2.3744i \times 10^{-4} \\ -6.2981i \times 10^{-5} & -2.6708i \times 10^{-4} & 0.117 + 0.4476i & 5.9273i \times 10^{-5} & 2.3744i \times 10^{-4} & 0.234 + 0.8952i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$Zdnew := \text{stack}(ZdA, ZdB)$

$$Zdnew = \begin{pmatrix} 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -1.1856i \times 10^{-4} & 0.117 + 0.4476i & -2.6708i \times 10^{-4} & -6.2981i \times 10^{-5} \\ -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -2.0784i \times 10^{-4} & 0.117 + 0.4476i & -2.6708i \times 10^{-4} \\ -1.1856i \times 10^{-4} & -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -5.5575i \times 10^{-5} & -2.0784i \times 10^{-4} & 0.117 + 0.4476i \\ 0.117 + 0.4476i & -2.0784i \times 10^{-4} & -5.5575i \times 10^{-5} & 0.234 + 0.8952i & 2.3744i \times 10^{-4} & 5.9273i \times 10^{-5} \\ -2.6708i \times 10^{-4} & 0.117 + 0.4476i & -2.0784i \times 10^{-4} & 2.3744i \times 10^{-4} & 0.234 + 0.8952i & 2.3744i \times 10^{-4} \\ -6.2981i \times 10^{-5} & -2.6708i \times 10^{-4} & 0.117 + 0.4476i & 5.9273i \times 10^{-5} & 2.3744i \times 10^{-4} & 0.234 + 0.8952i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$\begin{pmatrix} -6.29811\times 10 & -2.67081\times 10 & 0.117+0.447i & 5.92751\times 10 & 2.37441\times 10 & 0.234+0.8932i \end{pmatrix}$$

$$Zequiv := Za - Zbnew\cdot Zdnew^{-1}\cdot Zcnew$$

$$Zequiv = \begin{pmatrix} 0.1343+1.0626i & 0.0953+0.577i & 0.0953+0.493i \\ 0.0953+0.577i & 0.1343+1.0625i & 0.0953+0.577i \\ 0.0953+0.493i & 0.0953+0.577i & 0.1343+1.0626i \end{pmatrix}\frac{ohm}{mi}$$

Now if we had instead used the approximations of using the GMR of the bundle

$$Rsbundle := (Ds\cdot Da1a2\cdot Da1a3)^{\frac{1}{3}}$$

$$Lperlengthnew := Indcons\cdot\begin{pmatrix} \ln\left(\frac{De}{Rsbundle}\right) & \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Da1c1}\right) \\ \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Rsbundle}\right) & \ln\left(\frac{De}{Db1c1}\right) \\ \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Rsbundle}\right) \end{pmatrix}$$

The resistance matrix must also be modified since there are now parallel conductors

$$Rperlengthnew := \begin{pmatrix} \frac{Rac_{bund3}}{3}+Rd & Rd & Rd \\ Rd & \frac{Rac_{bund3}}{3}+Rd & Rd \\ Rd & Rd & \frac{Rac_{bund3}}{3}+Rd \end{pmatrix}$$

$$Zperlengthnew := Rperlengthnew + j \cdot 2 \cdot \pi \cdot freq \cdot Lperlengthnew$$

$$Zperlengthnew = \begin{pmatrix} 0.1343 + 1.0626i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.577i & 0.1343 + 1.0626i & 0.0953 + 0.577i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.1343 + 1.0626i \end{pmatrix} \frac{ohm}{mi}$$

Error between these methods:

$$Error := Zperlengthnew - Zequiv$$

$$Error = \begin{pmatrix} -1.967 \times 10^{-5} + 7.5221i \times 10^{-5} & -7.8588 \times 10^{-6} + 3.0065i \times 10^{-5} & 1.5731 \times 10^{-5} - 6.0149i \times 10^{-5} \\ -7.8588 \times 10^{-6} + 3.0065i \times 10^{-5} & -3.1441 \times 10^{-5} + 1.203i \times 10^{-4} & -7.8588 \times 10^{-6} + 3.0065i \times 10^{-5} \\ 1.5731 \times 10^{-5} - 6.0149i \times 10^{-5} & -7.8588 \times 10^{-6} + 3.0065i \times 10^{-5} & -1.967 \times 10^{-5} + 7.5221i \times 10^{-5} \end{pmatrix} \frac{ohm}{mi}$$

$$\overrightarrow{\left| \frac{Error}{Zequiv} \right|} = \begin{pmatrix} 7.2594 \times 10^{-3} & 5.3138 \times 10^{-3} & 0.0124 \\ 5.3138 \times 10^{-3} & 0.0116 & 5.3138 \times 10^{-3} \\ 0.0124 & 5.3138 \times 10^{-3} & 7.2594 \times 10^{-3} \end{pmatrix} \%$$