

## EE 523: Line Constants Example

### A. Resistance Matrix

AC Resistance from table:

$$R_{ac} := 0.278 \frac{\text{ohm}}{\text{mi}} \quad \text{at 25 C and} \quad \text{freq} := 60\text{Hz}$$

$$\text{CarsonsResistConst} := 9.869 \times 10^{-7} \frac{\text{ohm}}{\text{m}\cdot\text{Hz}}$$

$$R_d := \text{CarsonsResistConst} \cdot \text{freq} \qquad R_d = 0.0953 \frac{\text{ohm}}{\text{mi}}$$

$$R_{self} := R_{ac} + R_d \qquad R_{self} = 0.3733 \frac{\text{ohm}}{\text{mi}}$$

$$R_{perlength} := \begin{pmatrix} R_{self} & R_d & R_d \\ R_d & R_{self} & R_d \\ R_d & R_d & R_{self} \end{pmatrix}$$

$$R_{perlength} = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

### B. Inductance Matrix

$$\mu_0 := 4 \cdot \pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \qquad \rho := 100 \text{ohm}\cdot\text{m}$$

Calculate GMR from conductor diameter

$$\text{dia} := 0.528 \text{in} \qquad \text{GMR} := e^{\frac{-1}{4}} \cdot \frac{\text{dia}}{2} \qquad \text{GMR} = 0.01713 \text{ft}$$

Conductor GMR from table:

$D_s := 0.01668\text{ft}$  Note that this doesn't match calculation based on diameter, this is due to effects of stranding and skin effect.

$$D_e := 2160 \cdot \frac{\text{ft} \cdot \text{Hz}^{0.5}}{(\text{ohm} \cdot \text{m})^{0.5}}$$

$$D_{e_{\text{eff}}} := D_e \cdot \sqrt{\frac{\rho}{\text{freq}}} \quad D_e = 2.78855 \times 10^3 \text{ft}$$

$$\text{Indcons} := \frac{\mu_0}{2 \cdot \pi}$$

$$D_{ab} := 10\text{ft} \quad D_{ac} := 20\text{ft} \quad D_{bc} := 10\text{ft}$$

$$L_{\text{perlength}} := \begin{pmatrix} \text{Indcons} \cdot \ln\left(\frac{D_e}{D_s}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{ab}}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{ac}}\right) \\ \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{ab}}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_s}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{bc}}\right) \\ \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{ac}}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_{bc}}\right) & \text{Indcons} \cdot \ln\left(\frac{D_e}{D_s}\right) \end{pmatrix}$$

$$L_{\text{perlength}} = \begin{pmatrix} 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} & 1.58924 \times 10^{-3} \\ 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} \\ 1.58924 \times 10^{-3} & 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} \end{pmatrix} \left| \frac{\text{H}}{\text{mi}} \right.$$

$$Z_{\text{perlength}} := R_{\text{perlength}} + j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{perlength}}$$

$$Z_{\text{perlength}} = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \left| \frac{\text{ohm}}{\text{mi}} \right.$$

If length = 40 miles:

$$Z_{\text{line}} := Z_{\text{perlength}} \cdot 40 \text{mi}$$

$$Z_{\text{line}} = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{\text{line}} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.55548 + 110.79032i & 0.97119 - 0.56072i & -0.97119 - 0.56072i \\ -0.97119 - 0.56072i & 11.12 + 32.16612i & -1.94238 + 1.12144i \\ 0.97119 - 0.56072i & -1.94238 + 1.12144i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

By comparison:

$$D_m := (D_{ab} \cdot D_{bc} \cdot D_{ac})^{\frac{1}{3}}$$

$$Z_1 := \left( R_{ac} + j \cdot 2 \cdot \pi \cdot \text{freq} \cdot \text{Indcons} \cdot \ln \left( \frac{D_m}{D_s} \right) \right) \cdot 40 \text{mi}$$

$$Z_1 - Z_{012_{1,1}} = 0 \Omega$$

• **Transposition**

$$R_p := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 1:

$$f_1 := 0.2 \quad f_2 := 0.3 \quad f_3 := 0.5$$

$$Z_{net} := f_1 \cdot Z_{line} + f_2 \cdot R_p^{-1} \cdot Z_{line} \cdot R_p + f_3 \cdot R_p \cdot Z_{line} \cdot R_p^{-1}$$

$$Z_{net} = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 25.64735i & 3.81183 + 26.65664i \\ 3.81183 + 25.64735i & 14.93183 + 58.37419i & 3.81183 + 26.32021i \\ 3.81183 + 26.65664i & 3.81183 + 26.32021i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z_{0121} := A_{012}^{-1} \cdot Z_{net} \cdot A_{012}$$

$$Z_{0121} = \begin{pmatrix} 22.55548 + 110.79032i & -0.29136 - 0.05607i & 0.29136 - 0.05607i \\ 0.29136 - 0.05607i & 11.12 + 32.16612i & 0.58271 + 0.11214i \\ -0.29136 - 0.05607i & -0.58271 + 0.11214i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

Example 2:

$$f_{13} := 0.4 \quad f_{23} := 0.6 \quad f_{33} := 0.0$$

$$Z_{net3} := f_{13} \cdot Z_{line} + f_{23} \cdot R_p^{-1} \cdot Z_{line} \cdot R_p + f_{33} \cdot R_p \cdot Z_{line} \cdot R_p^{-1}$$

$$Z_{net3} = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 27.3295i & 3.81183 + 25.98378i \\ 3.81183 + 27.3295i & 14.93183 + 58.37419i & 3.81183 + 25.31092i \\ 3.81183 + 25.98378i & 3.81183 + 25.31092i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z_{0123} := A_{012}^{-1} \cdot Z_{net3} \cdot A_{012}$$

$$Z_{0123} = \begin{pmatrix} 22.55548 + 110.79032i & 0.38848 + 0.44857i & -0.38848 + 0.44857i \\ -0.38848 + 0.44857i & 11.12 + 32.16612i & -0.77695 - 0.89715i \\ 0.38848 + 0.44857i & 0.77695 - 0.89715i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

Example 3

$$f_{16} := \frac{1}{3} \quad f_{26} := \frac{1}{3} \quad f_{36} := \frac{1}{3}$$

$$Z_{net6} := f_{16} \cdot Z_{line} + f_{26} \cdot R_p^{-1} \cdot Z_{line} \cdot R_p + f_{36} \cdot R_p \cdot Z_{line} \cdot R_p^{-1}$$

$$Z_{net6} = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 26.20807i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 14.93183 + 58.37419i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 3.81183 + 26.20807i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z_{0126} := A_{012}^{-1} \cdot Z_{net6} \cdot A_{012}$$

$$Z_{0126} = \begin{pmatrix} 22.555 + 110.79i & 0 & 0 \\ 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$

Measure of imbalance

$$\text{define unit: } mS := \frac{S}{1000}$$

Original System

$$Y_{012} := Z_{012}^{-1}$$

$$Y_{012} = \begin{pmatrix} 1.7687 - 8.67132i & 0.31072 + 0.01039i & -0.14637 - 0.27428i \\ -0.14637 - 0.27428i & 9.71663 - 27.84551i & -1.93273 - 0.28446i \\ 0.31072 + 0.01039i & 0.72001 + 1.81603i & 9.71663 - 27.84551i \end{pmatrix} mS$$

First transposed case:

$$Y_{0121} := Z_{0121}^{-1}$$

$$Y_{0121} = \begin{pmatrix} 1.76473 - 8.66714i & -0.05715 - 0.05228i & 0.07389 + 0.02557i \\ 0.07389 + 0.02557i & 9.6082 - 27.77506i & 0.33557 + 0.38762i \\ -0.05715 - 0.05228i & -0.45617 - 0.2345i & 9.6082 - 27.77506i \end{pmatrix} \text{ mS}$$

Second transposed case:

$$Y_{0123} := Z_{0123}^{-1}$$

$$Y_{0123} = \begin{pmatrix} 1.76552 - 8.66803i & 0.02348 + 0.14835i & -0.14356 + 0.0535i \\ -0.14356 + 0.0535i & 9.63221 - 27.79083i & -0.04638 - 1.02282i \\ 0.02348 + 0.14835i & 1.00575 - 0.19655i & 9.63221 - 27.79083i \end{pmatrix} \text{ mS}$$

Fully Transposed Case

$$Y_{0126} := Z_{0126}^{-1}$$

$$Y_{0126} = \begin{pmatrix} 1.7645 - 8.6668i & 0 & 0 \\ 0 & 9.6002 - 27.7698i & 0 \\ 0 & 0 & 9.6002 - 27.7698i \end{pmatrix} \text{ mS}$$

• **Twisting**

$$T_{23} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_{23}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_{13} := \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T_{13}^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$T_{12} := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_{12}^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example 4:

$$\underline{f1} := 0 \quad \underline{f2} := \frac{1}{3} \quad \underline{f3} := \frac{1}{3} \quad \underline{f4} := \frac{1}{3}$$

$$Z_{net4} := \underline{f1} \cdot Z_{line} + \underline{f2} \cdot T_{23} \cdot Z_{line} \cdot T_{23} + \underline{f3} \cdot T_{13} \cdot Z_{line} \cdot T_{13} + \underline{f4} \cdot T_{12} \cdot Z_{line} \cdot T_{12}$$

$$Z_{net4} = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 26.20807i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 14.93183 + 58.37419i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 3.81183 + 26.20807i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z_{0124} := A_{012}^{-1} \cdot Z_{net4} \cdot A_{012}$$

$$Z_{0124} = \begin{pmatrix} 22.55548 + 110.79032i & 0 & 0 \\ 0 & 11.12 + 32.16612i & 0 \\ 0 & 0 & 11.12 + 32.16612i \end{pmatrix} \Omega$$

Example 5:

$$\underline{f1} := \frac{1}{2} \quad \underline{f2} := \frac{1}{2} \quad \underline{f3} := 0 \quad \underline{f4} := 0$$

$$Z_{net5} := f1 \cdot Z_{line} + f2 \cdot T_{23} \cdot Z_{line} \cdot T_{23} + f3 \cdot T_{13} \cdot Z_{line} \cdot T_{13} + f4 \cdot T_{12} \cdot Z_{line} \cdot T_{12}$$

$$Z_{net5} = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 25.64735i & 3.81183 + 25.64735i \\ 3.81183 + 25.64735i & 14.93183 + 58.37419i & 3.81183 + 27.3295i \\ 3.81183 + 25.64735i & 3.81183 + 27.3295i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z_{0125} := A_{012}^{-1} \cdot Z_{net5} \cdot A_{012}$$

$$Z_{0125} = \begin{pmatrix} 22.55548 + 110.79032i & -0.56072i & -0.56072i \\ -0.56072i & 11.12 + 32.16612i & 1.12144i \\ -0.56072i & 1.12144i & 11.12 + 32.16612i \end{pmatrix} \Omega$$