EE 523: Line Constants Example

A. Resistance Matrix

AC Resistance from table:

$Rac := 0.278\frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$CarsonsResistConst := 9.869 \times 10^{-7}\frac{ohm}{m \cdot Hz}$

$Rd := CarsonsResistConst \cdot freq$ $\quad Rd = 0.0953\frac{ohm}{mi}$

$Rself := Rac + Rd$ $\quad Rself = 0.3733\frac{ohm}{mi}$

$$Rperlength := \begin{pmatrix} Rself & Rd & Rd \\ Rd & Rself & Rd \\ Rd & Rd & Rself \end{pmatrix}$$

$$Rperlength = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 \end{pmatrix}\frac{ohm}{mi}$$

B. Inductance Matrix

$\mu o := 4 \cdot \pi \cdot 10^{-7}\frac{H}{m}$ $\quad \rho := 100ohm \cdot m$

Calculate GMR from conductor diameter

$dia := 0.528in$ $\quad GMR := e^{\frac{-1}{4}} \cdot \frac{dia}{2}$ $\quad GMR = 0.01713\,ft$

Conductor GMR from table:

$Ds := 0.01668\text{ft}$ Note that this doesn't match calculation based on diameter, this is due to effects of stranding and skin effect.

$$De := 2160 \cdot \frac{\text{ft} \cdot \text{Hz}^{0.5}}{(\text{ohm} \cdot \text{m})^{0.5}}$$

$$De := De \cdot \sqrt{\frac{\rho}{freq}} \qquad De = 2.78855 \times 10^{3}\,\text{ft}$$

$$Indcons := \frac{\mu o}{2 \cdot \pi}$$

$Dab := 10\text{ft}$ $Dac := 20\text{ft}$ $Dbc := 10\text{ft}$

$$Lperlength := \begin{pmatrix} Indcons \cdot \ln\left(\frac{De}{Ds}\right) & Indcons \cdot \ln\left(\frac{De}{Dab}\right) & Indcons \cdot \ln\left(\frac{De}{Dac}\right) \\ Indcons \cdot \ln\left(\frac{De}{Dab}\right) & Indcons \cdot \ln\left(\frac{De}{Ds}\right) & Indcons \cdot \ln\left(\frac{De}{Dbc}\right) \\ Indcons \cdot \ln\left(\frac{De}{Dac}\right) & Indcons \cdot \ln\left(\frac{De}{Dbc}\right) & Indcons \cdot \ln\left(\frac{De}{Ds}\right) \end{pmatrix}$$

$$Lperlength = \begin{pmatrix} 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} & 1.58924 \times 10^{-3} \\ 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} \\ 1.58924 \times 10^{-3} & 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} \end{pmatrix} \frac{\text{H}}{\text{mi}}$$

$$Zperlength := Rperlength + j \cdot 2 \cdot \pi \cdot freq \cdot Lperlength$$

$$Zperlength = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

If length = 40 miles:

$$Zline := Zperlength \cdot 40mi$$

$$Zline = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$Z012 := A_{012}^{-1} \cdot Zline \cdot A_{012}$$

$$Z012 = \begin{pmatrix} 22.55548 + 110.79032i & 0.97119 - 0.56072i & -0.97119 - 0.56072i \\ -0.97119 - 0.56072i & 11.12 + 32.16612i & -1.94238 + 1.12144i \\ 0.97119 - 0.56072i & 1.94238 + 1.12144i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

By comparsion:

$$Dm := (Dab \cdot Dbc \cdot Dac)^{\frac{1}{3}}$$

$$Z_1 := \left(Rac + j \cdot 2 \cdot \pi \cdot freq \cdot Indcons \cdot \ln\left(\frac{Dm}{Ds}\right)\right) \cdot 40mi$$

$$Z_1 - Z012_{1,1} = 0\Omega$$

- ***Transposition***

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 1:

$$f1 := 0.2 \quad f2 := 0.3 \quad f3 := 0.5$$

$$Znet := f1 \cdot Zline + f2 \cdot Rp^{-1} \cdot Zline \cdot Rp + f3 \cdot Rp \cdot Zline \cdot Rp^{-1}$$

$$Znet = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 25.64735i & 3.81183 + 26.65664i \\ 3.81183 + 25.64735i & 14.93183 + 58.37419i & 3.81183 + 26.32021i \\ 3.81183 + 26.65664i & 3.81183 + 26.32021i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z0121 := A_{012}^{-1} \cdot Znet \cdot A_{012}$$

$$Z0121 = \begin{pmatrix} 22.55548 + 110.79032i & -0.29136 - 0.05607i & 0.29136 - 0.05607i \\ 0.29136 - 0.05607i & 11.12 + 32.16612i & 0.58271 + 0.11214i \\ -0.29136 - 0.05607i & -0.58271 + 0.11214i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

Example 2:

$$f13 := 0.4 \quad f23 := 0.6 \quad f33 := 0.0$$

$$Znet3 := f13 \cdot Zline + f23 \cdot Rp^{-1} \cdot Zline \cdot Rp + f33 \cdot Rp \cdot Zline \cdot Rp^{-1}$$

$$Znet3 = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 27.3295i & 3.81183 + 25.98378i \\ 3.81183 + 27.3295i & 14.93183 + 58.37419i & 3.81183 + 25.31092i \\ 3.81183 + 25.98378i & 3.81183 + 25.31092i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z0123 := A_{012}^{-1} \cdot Znet3 \cdot A_{012}$$

$$Z0123 = \begin{pmatrix} 22.55548 + 110.79032i & 0.38848 + 0.44857i & -0.38848 + 0.44857i \\ -0.38848 + 0.44857i & 11.12 + 32.16612i & -0.77695 - 0.89715i \\ 0.38848 + 0.44857i & 0.77695 - 0.89715i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

Example 3

$$f16 := \frac{1}{3} \quad f26 := \frac{1}{3} \quad f36 := \frac{1}{3}$$

$$Znet6 := f16 \cdot Zline + f26 \cdot Rp^{-1} \cdot Zline \cdot Rp + f36 \cdot Rp \cdot Zline \cdot Rp^{-1}$$

$$Znet6 = \begin{pmatrix} 14.93183 + 58.37419i & 3.81183 + 26.20807i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 14.93183 + 58.37419i & 3.81183 + 26.20807i \\ 3.81183 + 26.20807i & 3.81183 + 26.20807i & 14.93183 + 58.37419i \end{pmatrix} \Omega$$

$$Z0126 := A_{012}^{-1} \cdot Znet6 \cdot A_{012}$$

$$Z0126 = \begin{pmatrix} 22.555 + 110.79i & 0 & 0 \\ 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$

Measure of imbalance

define unit: $mS := \frac{S}{1000}$

Original System

$$Y012 := Z012^{-1}$$

$$Y012 = \begin{pmatrix} 1.7687 - 8.67132i & 0.31072 + 0.01039i & -0.14637 - 0.27428i \\ -0.14637 - 0.27428i & 9.71663 - 27.84551i & -1.93273 - 0.28446i \\ 0.31072 + 0.01039i & 0.72001 + 1.81603i & 9.71663 - 27.84551i \end{pmatrix} mS$$

First transposed case:

$$Y0121 := Z0121^{-1}$$

$$Y0121 = \begin{pmatrix} 1.76473 - 8.66714i & -0.05715 - 0.05228i & 0.07389 + 0.02557i \\ 0.07389 + 0.02557i & 9.6082 - 27.77506i & 0.33557 + 0.38762i \\ -0.05715 - 0.05228i & -0.45617 - 0.2345i & 9.6082 - 27.77506i \end{pmatrix} \text{mS}$$

Second transposed case:

$$Y0123 := Z0123^{-1}$$

$$Y0123 = \begin{pmatrix} 1.76552 - 8.66803i & 0.02348 + 0.14835i & -0.14356 + 0.0535i \\ -0.14356 + 0.0535i & 9.63221 - 27.79083i & -0.04638 - 1.02282i \\ 0.02348 + 0.14835i & 1.00575 - 0.19655i & 9.63221 - 27.79083i \end{pmatrix} \text{mS}$$

Fully Transposed Case

$$Y0126 := Z0126^{-1}$$

$$Y0126 = \begin{pmatrix} 1.7645 - 8.6668i & 0 & 0 \\ 0 & 9.6002 - 27.7698i & 0 \\ 0 & 0 & 9.6002 - 27.7698i \end{pmatrix} \text{mS}$$