# EE 523: Line Constants Example

## A. Resistance Matrix

AC Resistance from table:

$Rac := 0.278 \frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$CarsonsResistConst := 9.869 \times 10^{-7} \frac{ohm}{m \cdot Hz}$

$Rd := CarsonsResistConst \cdot freq$ $\quad Rd = 0.0953 \frac{ohm}{mi}$

$Rself := Rac + Rd$ $\quad Rself = 0.3733 \frac{ohm}{mi}$

$$Rperlength := \begin{pmatrix} Rself & Rd & Rd \\ Rd & Rself & Rd \\ Rd & Rd & Rself \end{pmatrix}$$

$$Rperlength = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 \end{pmatrix} \frac{ohm}{mi}$$

## B. Inductance Matrix

$\mu o := 4 \cdot \pi \cdot 10^{-7} \frac{H}{m}$ $\quad \rho := 100 ohm \cdot m$

Calculate GMR from conductor diameter

$dia := 0.528in$ $\quad GMR := e^{\frac{-1}{4}} \cdot \frac{dia}{2}$ $\quad GMR = 0.01713\,ft$

Conductor GMR from table:

$Ds := 0.01668ft$ Note that this doesn't match calculation based on diameter, this is due to effects of stranding.

$$De := 2160 \cdot \frac{ft \cdot Hz^{0.5}}{(ohm \cdot m)^{0.5}}$$

$$De := De \cdot \sqrt{\frac{\rho}{freq}} \qquad De = 2.78855 \times 10^{3}\,ft$$

$$Indcons := \frac{\mu o}{2 \cdot \pi}$$

$Dab := 10ft$ $Dac := 20ft$ $Dbc := 10ft$

$$Lperlength := \begin{pmatrix} Indcons \cdot \ln\left(\frac{De}{Ds}\right) & Indcons \cdot \ln\left(\frac{De}{Dab}\right) & Indcons \cdot \ln\left(\frac{De}{Dac}\right) \\ Indcons \cdot \ln\left(\frac{De}{Dab}\right) & Indcons \cdot \ln\left(\frac{De}{Ds}\right) & Indcons \cdot \ln\left(\frac{De}{Dbc}\right) \\ Indcons \cdot \ln\left(\frac{De}{Dac}\right) & Indcons \cdot \ln\left(\frac{De}{Dbc}\right) & Indcons \cdot \ln\left(\frac{De}{Ds}\right) \end{pmatrix}$$

$$Lperlength = \begin{pmatrix} 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} & 1.58924 \times 10^{-3} \\ 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} & 1.81234 \times 10^{-3} \\ 1.58924 \times 10^{-3} & 1.81234 \times 10^{-3} & 3.87106 \times 10^{-3} \end{pmatrix} \frac{H}{mi}$$

$$Zperlength := Rperlength + j \cdot 2 \cdot \pi \cdot freq \cdot Lperlength$$

$$Zperlength = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \frac{ohm}{mi}$$

If length = 40 miles:

$$Zline := Zperlength \cdot 40mi$$

$$Zline = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$Z012 := A_{012}{}^{-1} \cdot Zline \cdot A_{012}$$

$$Z012 = \begin{pmatrix} 22.55548 + 110.79032i & 0.97119 - 0.56072i & -0.97119 - 0.56072i \\ -0.97119 - 0.56072i & 11.12 + 32.16612i & -1.94238 + 1.12144i \\ 0.97119 - 0.56072i & 1.94238 + 1.12144i & 11.12 + 32.16612i \end{pmatrix} \Omega$$

By comparsion:

$$Dm := (Dab \cdot Dbc \cdot Dac)^{\frac{1}{3}}$$

$$Z_1 := \left( Rac + j \cdot 2 \cdot \pi \cdot freq \cdot Indcons \cdot \ln\left(\frac{Dm}{Ds}\right)\right) \cdot 40mi$$

$$Z_1 - Z012_{1,1} = 0\Omega$$