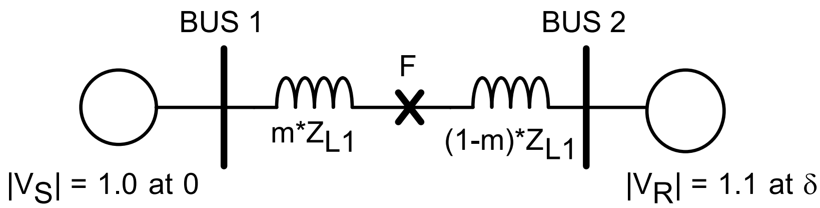
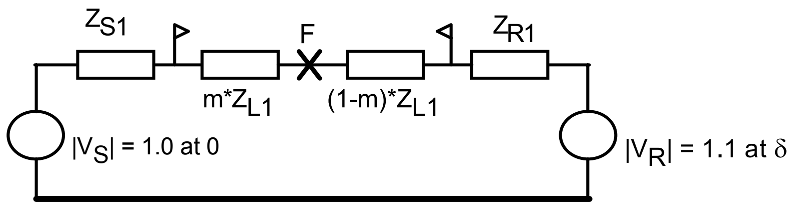
Fault Calculations with Power Flow

- The system below has a prefault power flow condition due to the angle and magnitude differences between the sources.
- The fault calculations need to change a little to ensure that the positive sequence current reflects this power flow in the case of a fault where power flow can continue to flow.
- Lets look at a SLG fault case.



- The negative and zero sequence circuits will be the same as one would in a case where the sources have equal angles and magnitudes, so they will not be described here.
- Positive seequence equivalent circuit:



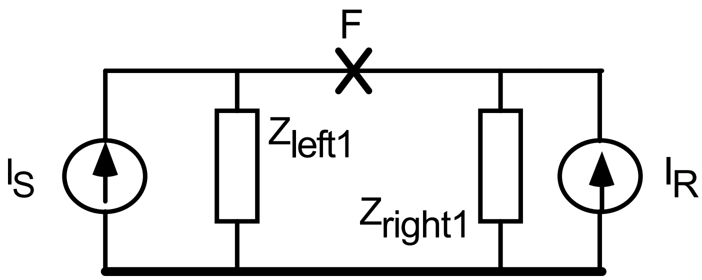
- There are effectively two components to the current seen at each relay, anad they can be determined using superposition.
  1. The fault current that flows due to the fault and leave this network at point F and reenters from the neutral plane.
  2. The current that flows between the two sources, the load current.

1. Determining fault current

- We need to find a Thevenin equivalent circuit.
- The process is actually a standard circuit analysis approach (Millman's Theorem), but is typically avoided if the voltage sources are all assumed to have the same magnitude and angle.
  1. Convert the two sources to their Norton equivalents, using the impedance between the source and the fault point. Note that these are phasor calculations.

$$Z_{left1} = Z_{S1} + m \cdot Z_{L1} \qquad Z_{right1} = Z_{R1} + (1 - m) \cdot Z_{L1}$$

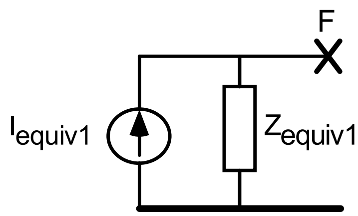
$$I_{Norton\_left} = \frac{V_{S1}}{Z_{left1}} \qquad I_{Norton\_left} = \frac{V_{S1}}{Z_{left1}}$$



2. Note that the impedances are in parallel and the current soruces are effectively in parallel
   a. Combine the impedances in parallel
   b. Combine the two current sources (note that this is not limited to two sources)

$$Z_{equiv1} = \left(\frac{1}{Z_{left1}} + \frac{1}{Z_{right1}}\right)^{-1}$$

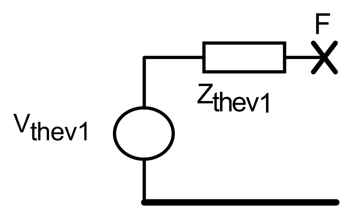
$$I_{equiv1} = I_{Norton\_left} + I_{Norton\_right}$$



   c. Then convert back to a Thevenin equivalent

$$Z_{thev1} = Z_{equiv1}$$

$$V_{thev1} = I_{equiv1} \cdot Z_{equiv1}$$



- This Thevenin equivalent source is used for the fault calculations. ***But not for the power flow calculation***
  1. Note that the Thevenin impedance is the same as we always do.
  2. Now the voltage source has a magntiude and angle that refelcts the difference between the two sources.
  3. If the sources both have the same magnitude and angle, the resulting Thevenin voltage source will match that.

2. Determining power flow current

- This is just like any other power flow calculation. In this case you can look between the two known source voltages and the ***total impedance*** between them. In other cases you might need to find V1 and V2 and just use the line impedance.

$$I_{12} = \frac{V_{S1} - V_{R1}}{Z_{S1} + Z_{L1} + Z_{R1}}$$

$$I_{21} = \frac{V_{R1} - V_{S1}}{\left(Z_{S1} + Z_{L1} + Z_{R1}\right)}$$

- Notes:
  1. The fault location doesn't matter in this calculation
  2. The Thevenin equivalent source from above is not used
  3. I_{12} flows in the opposite direction I_{21}

3. Total sequence currents

- The positive sequence current for the relay at bus 1 (phasor sums):

$$I_{Relay1} = I_{f\_relay1} + I_{12\_relay1}$$

$$I_{Relay2} = I_{f\_relay2} - I_{12\_relay1}$$

  - I_{f_relay1} and I_{f_relay2} come from current dividers as usual

- The negative and zero sequence currents do not include an load flow current and are simply from current dividers from the fault calculation.

Example

$pu := 1 \quad MVA := 1000kW \quad S_{base} := 100MVA \quad kV_{base} := 500kV$

$a := 1 \cdot e^{j \cdot 120deg} \qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

- System parameters

$ZsL1 := (0.0006 + j \cdot 0.02079)pu \qquad ZrL1 := (0.00067 + j \cdot 0.01633)pu \qquad ZrL2 := ZrL1$

$ZsL0 := (0.0021 + j \cdot 0.02037)pu \qquad ZrL0 := (0.00065 + j \cdot 0.01429)pu$

$ZsL2 := ZsL1 \qquad ZL1 := (0.00205 + j \cdot 0.02893)pu \qquad ZL2 := ZL1$

$ZL0 := (0.01708 + j \cdot 0.10764)pu$

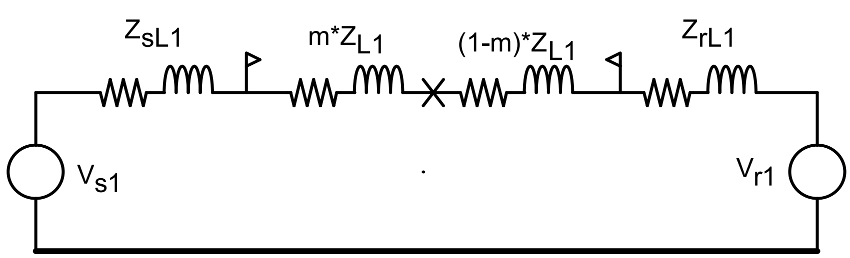
$Z_{base} := \frac{kV_{base}^2}{S_{base}} \qquad Z_{base} = 2500\,\Omega$

$V_{S1} := \left(\frac{500kV}{kV_{base}}\right) e^{j \cdot 0deg} \qquad V_{R1} := \left(\frac{500kV}{kV_{base}}\right) e^{-j \cdot 30deg}$

- Now we need to find the Thevenin equivalent with the unbalanced sources included.

Fault location: $M := 0.3$

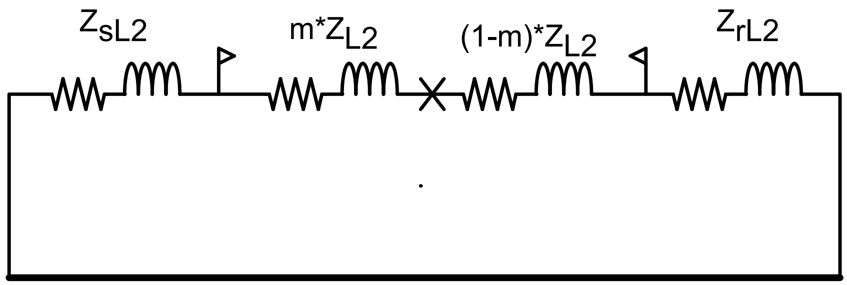
Sequence equivalent circuits:



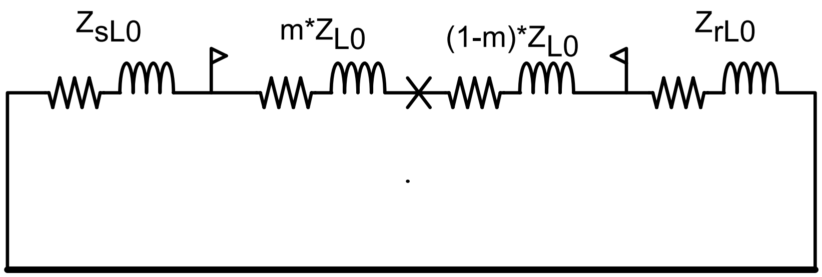
$M := 0.3$

$$Z_{thev1} := \left[\frac{1}{ZsL1 + M \cdot ZL1} + \frac{1}{ZrL1 + (1 - M) \cdot ZL1} \right]^{-1}$$

$\left|Z_{thev1}\right| = 0.0163 \qquad \arg\left(Z_{thev1}\right) = 87.2232\,\text{deg}$



$Z_{thev2} := Z_{thev1}$



$$Z_{thev0} := \left[\frac{1}{ZsL0 + M\cdot ZL0} + \frac{1}{ZrL0 + (1-M)\cdot ZL0}\right]^{-1}$$

$\left|Z_{thev0}\right| = 0.0335 \qquad \arg\left(Z_{thev0}\right) = 82.1172\,\text{deg}$

$$I_{s1Nor} := \frac{V_{S1}}{ZsL1 + M\cdot ZL1} \qquad I_{r1Nor} := \frac{V_{R1}}{ZrL1 + (1-M)\cdot ZL1}$$

$I_{1\_Nor} := I_{s1Nor} + I_{r1Nor}$

$V_{1\_Thev} := I_{1\_Nor}\cdot Z_{thev1} \qquad \left|V_{1\_Thev}\right| = 0.9684\,\text{pu} \quad \arg\left(V_{1\_Thev}\right) = -13.3447\,\text{deg}$

$$I_{f0}\left(R_f\right) := \frac{V_{1\_Thev}}{Z_{thev0} + Z_{thev1} + Z_{thev2} + 3\cdot R_f}$$

$I_{f1}\left(R_f\right) := I_{f0}\left(R_f\right)$

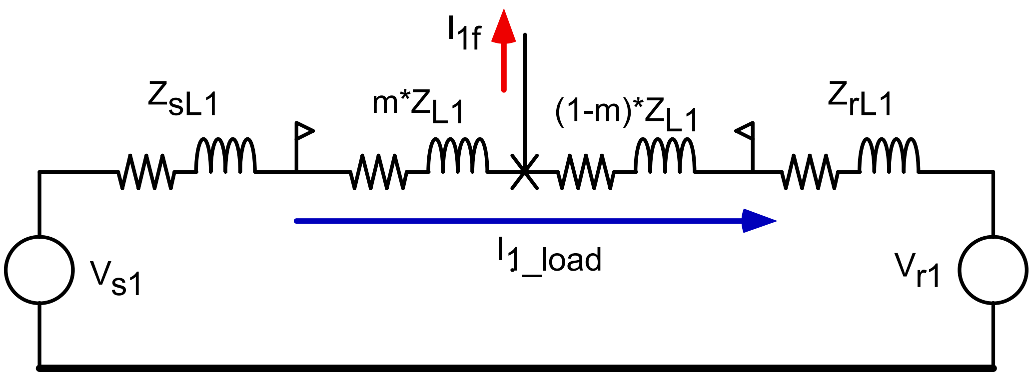
$I_{f2}\left(R_f\right) := I_{f0}\left(R_f\right)$

- Voltages and Currents at Relay A:

$$I_{fA1}(R_f) := I_{f1}(R_f)\cdot\left[\frac{ZrL1 + (1-M)\cdot ZL1}{(ZsL1 + M\cdot ZL1) + [\,ZrL1 + (1-M)\cdot ZL1\,]}\right]$$

$$|I_{fA1}(0pu)| = 8.1165 \qquad \arg(I_{fA1}(0pu)) = -98.3996\,deg$$

However, the positive sequence current seen by the relay will include the load current



$$I_{1\_load} := \frac{V_{S1} - V_{R1}}{ZsL1 + ZL1 + ZrL1} \qquad \arg(V_{R1}) = -30\,deg$$

$$|I_{1\_load}| = 7.8272 \quad \cdot g(I_{1\_load}) = -12.1225\,deg \qquad I_{base} := \frac{S_{base}}{\sqrt{3}\cdot kV_{base}}$$

$$|I_{1\_load}|\cdot I_{base} = 903.8052\,A$$

$$I_{relayA1}(R_f) := I_{fA1}(R_f) + I_{1\_load}$$

$$I_{relayA1}(0pu) = 6.467 - 9.6732i \qquad |I_{relayA1}(0pu)| = 11.6358$$

$$\arg(I_{relayA1}(0pu)) = -56.2352\,deg$$

$$|I_{relayA1}(0pu)|\cdot I_{base} = 1.3436\times 10^{3}\,A$$

$$I_{relayA2}(R_f) := I_{f2}(R_f)\cdot\left[\frac{ZrL2 + (1-M)\cdot ZL2}{(ZsL2 + M\cdot ZL2) + [\,ZrL2 + (1-M)\cdot ZL2\,]}\right]$$

$$|I_{relayA2}(0pu)| = 8.1165 \qquad \arg(I_{relayA2}(0pu)) = -98.3996\,deg$$

No negative or zero sequence load flow

$$I_{relayA0}(R_f) := I_{f0}(R_f)\cdot\left[\frac{ZrL0 + (1-M)\cdot ZL0}{(ZsL0 + M\cdot ZL0) + [ZrL0 + (1-M)\cdot ZL0]}\right]$$

$$\left|I_{relayA0}(0pu)\right| = 9.2295 \qquad \arg\left(I_{relayA0}(0pu)\right) = -98.0557\,deg$$

$$V_{relayA1}(R_f) := V_{S1} - I_{relayA1}(R_f)\cdot(ZsL1) \qquad \left|V_{relayA1}(0pu)\right| = 0.8054\,pu$$

$$\arg\left(V_{relayA1}(0pu)\right) = -9.1916\,deg$$

$$V_{relayA2}(R_f) := -I_{relayA2}(R_f)\cdot(ZsL2) \qquad \left|V_{relayA2}(0pu)\right| = 0.1688\,pu$$

$$\arg\left(V_{relayA2}(0pu)\right) = 169.9473\,deg$$

$$V_{relayA0}(R_f) := -I_{relayA0}(R_f)\cdot(ZsL0) \qquad \left|V_{relayA0}(0pu)\right| = 0.189\,pu$$

$$\arg\left(V_{relayA0}(0pu)\right) = 166.0583\,deg$$

$$I_{ABC\_RA}(R_f) := A_{012}\cdot\begin{pmatrix} I_{relayA0}(R_f) \\ I_{relayA1}(R_f) \\ I_{relayA2}(R_f) \end{pmatrix} \qquad V_{ABC\_RA}(R_f) := A_{012}\cdot\begin{pmatrix} V_{relayA0}(R_f) \\ V_{relayA1}(R_f) \\ V_{relayA2}(R_f) \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_RA}(0pu)\right|} = \begin{pmatrix} 27.1357 \\ 8.7473 \\ 6.8192 \end{pmatrix} pu \qquad \overrightarrow{\arg\left(I_{ABC\_RA}(0pu)\right)} = \begin{pmatrix} -81.5489 \\ -127.7695 \\ 111.6023 \end{pmatrix} deg$$

$$\overrightarrow{\left|V_{ABC\_RA}(0pu)\right|} = \begin{pmatrix} 0.4486 \\ 0.9729 \\ 0.9953 \end{pmatrix} pu \qquad \overrightarrow{\arg\left(V_{ABC\_RA}(0pu)\right)} = \begin{pmatrix} -6.8679 \\ -130.7249 \\ 111.26 \end{pmatrix} deg$$

$$\overrightarrow{\left|I_{ABC\_RA}(0pu)\right|}\cdot I_{base} = \begin{pmatrix} 3133.359 \\ 1010.047 \\ 787.413 \end{pmatrix} A \qquad \overrightarrow{\left|V_{ABC\_RA}(0pu)\right|}\cdot\frac{kV_{base}}{\sqrt{3}} = \begin{pmatrix} 129.4936 \\ 280.8579 \\ 287.3233 \end{pmatrix} kV$$

- Voltages and Currents at Relay B:

$$I_{fB1}(R_f) := I_{f1}(R_f)\cdot\left[\frac{ZsL1 + M\cdot ZL1}{(ZsL1 + M\cdot ZL1) + [\,ZrL1 + (1-M)\cdot ZL1\,]}\right]$$

$$\left|I_{fB1}(0pu)\right| = 6.5333 \qquad \arg\left(I_{fB1}(0pu)\right) = -97.4672\,deg$$

Again, we need to incorporate load flow. In this case it will be subtracted from the relay current due to the polarity of the load current compared to the fault current

$$I_{relayB1}(R_f) := I_{fB1}(R_f) - I_{1\_load} \qquad \left|I_{relayB1}(0pu)\right| = 9.78\,pu$$

$$\arg\left(I_{relayB1}(0pu)\right) = -150.3772\,deg$$

$$I_{relayB2}(R_f) := I_{f2}(R_f)\cdot\left[\frac{ZsL2 + M\cdot ZL2}{(ZsL2 + M\cdot ZL2) + [\,ZrL2 + (1-M)\cdot ZL2\,]}\right]$$

$$\left|I_{relayB2}(0pu)\right| = 6.5333 \qquad \arg\left(I_{relayB2}(0pu)\right) = -97.4672\,deg$$

$$I_{relayB0}(R_f) := I_{f0}(R_f)\cdot\left[\frac{ZsL0 + M\cdot ZL0}{(ZsL0 + M\cdot ZL0) + [\,ZrL0 + (1-M)\cdot ZL0\,]}\right]$$

$$\left|I_{relayB0}(0pu)\right| = 5.4198 \qquad \arg\left(I_{relayB0}(0pu)\right) = -97.8614\,deg$$

$$V_{relayB1}(R_f) := V_{R1} - I_{relayB1}(R_f)\cdot(ZrL1) \qquad \left|V_{relayB1}(0pu)\right| = 0.8698\,pu$$

$$\arg\left(V_{relayB1}(0pu)\right) = -24.2985\,deg$$

$$V_{relayB2}(R_f) := -I_{relayB2}(R_f)\cdot(ZrL2) \qquad \left|V_{relayB2}(0pu)\right| = 0.1068\,pu$$

$$\arg\left(V_{relayB2}(0pu)\right) = 170.1833\,deg$$

$$V_{relayB0}(R_f) := -I_{relayB0}(R_f)\cdot(ZrL0) \qquad \left|V_{relayB0}(0pu)\right| = 0.0775\,pu$$

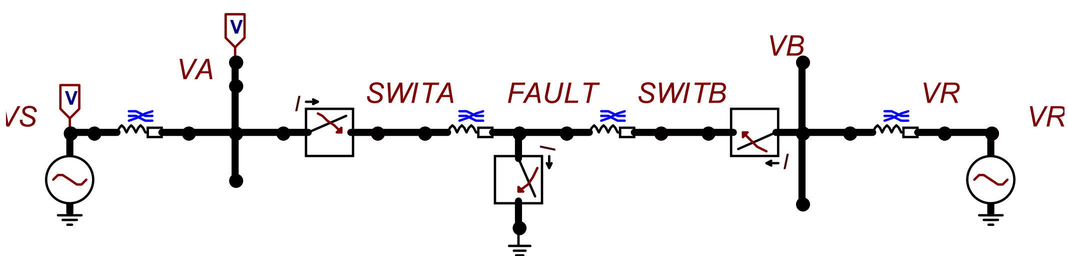
$$\arg\left(V_{relayB0}(0pu)\right) = 169.5342\,deg$$

$$I_{ABC\_RB}(R_f) := A_{012} \cdot \begin{pmatrix} I_{relayB0}(R_f) \\ I_{relayB1}(R_f) \\ I_{relayB2}(R_f) \end{pmatrix} \qquad V_{ABC\_RB}(R_f) := A_{012} \cdot \begin{pmatrix} V_{relayB0}(R_f) \\ V_{relayB1}(R_f) \\ V_{relayB2}(R_f) \end{pmatrix}$$

$$\left|\overrightarrow{I_{ABC\_RB}(0pu)}\right| = \begin{pmatrix} 19.4961 \\ 8.7473 \\ 6.8192 \end{pmatrix} pu \qquad \arg\left(\overrightarrow{I_{ABC\_RB}(0pu)}\right) = \begin{pmatrix} -121.1744 \\ 52.2305 \\ -68.3977 \end{pmatrix} deg$$

$$\left|\overrightarrow{V_{ABC\_RB}(0pu)}\right| = \begin{pmatrix} 0.6926 \\ 0.9533 \\ 0.9662 \end{pmatrix} pu \qquad \arg\left(\overrightarrow{V_{ABC\_RB}(0pu)}\right) = \begin{pmatrix} -28.0432 \\ -141.4748 \\ 95.5995 \end{pmatrix} deg$$

$$\left|\overrightarrow{I_{ABC\_RB}(0pu)}\right| \cdot I_{base} = \begin{pmatrix} 2251.22 \\ 1010.05 \\ 787.41 \end{pmatrix} A \qquad \left|\overrightarrow{V_{ABC\_RB}(0pu)}\right| \cdot \frac{kV_{base}}{\sqrt{3}} = \begin{pmatrix} 199.9506 \\ 275.1805 \\ 278.9299 \end{pmatrix} kV$$



$V_{A\_A} := 129.7kV$ at $-6.2deg$

$V_{A\_B} := 280.8kV$ at $-130.1deg$

$V_{A\_C} := 286.9kV$ at $112.0deg$

$I_{A\_A} := 3127A$ at $-80.8deg$ $\qquad$ $I_{B\_A} := 2249A$ at $-120.6deg$

$I_{A\_B} := 1009A$ at $-127.2deg$ $\qquad$ $I_{B\_B} := 1009A$ at $52.84deg$

$I_{A\_C} := 786.2kV$ at $112.4deg$ $\qquad$ $I_{B\_C} := 786.2kV$ at $-67.6deg$