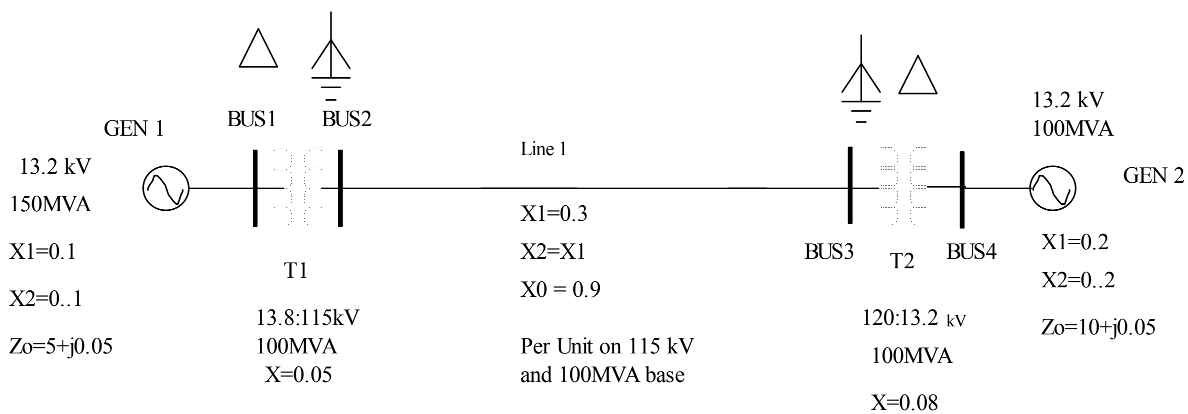
Zbus fault analysis example

Define units: $MVA := 1000kW$

$MW := MVA$ $MVAr := MVA$ $pu := 1$

$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$ $A1 := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$ $A_{012} := A1$

System Description:



Set bases:

$Sb := 100MVA$ $Vb2 := 115kV$ Use the line voltage as the reference

$Vb1 := Vb2 \cdot \frac{13.8kV}{115kV}$ $Vb1 = 13.8\,kV$ $Vb3 := Vb2 \cdot \frac{13.2kV}{120kV}$ $Vb3 = 12.65\,kV$

Change of Base Calculations:

Generator 1: $Sg1 := 150MVA$ $XG11old := 0.1pu$

$Vg1 := 13.2kV$

$XG11 := XG11old\cdot\left(\frac{Vg1}{Vb1}\right)^2\cdot\left(\frac{Sb}{Sg1}\right)$ $XG11 = 0.061\,pu$

$XG12 := XG11$ $XG12 = 0.061\,pu$

$ZG10old := 5.0 + j\cdot 0.05pu$

$ZG10 := ZG10old\cdot\left(\frac{Vg1}{Vb1}\right)^2\cdot\left(\frac{Sb}{Sg1}\right)$ $ZG10 = (3.0498 + 0.0305i)\,pu$

Transformer 1: $XT1old := 0.05pu$

$VT1old := 13.8kV$ $ST1 := 100MVA$

$XT11 := XT1old\cdot\left(\frac{VT1old}{Vb1}\right)^2\cdot\left(\frac{Sb}{ST1}\right)$ $XT11 = 0.05\,pu$

$XT12 := XT11$ $XT12 = 0.05\,pu$

$XT10 := XT11$ $XT10 = 0.05\,pu$

Transmisson Line: $XL1old := 0.3pu$ $XL0old := 0.9pu$

$VL1old := 115kV$ $SLine := 100MVA$

$XL11 := XL1old\cdot\left(\frac{VL1old}{Vb2}\right)^2\cdot\left(\frac{Sb}{SLine}\right)$ $XL11 = 0.3\,pu$

$$XL12 := XL11 \qquad XL12 = 0.3\,pu$$

$$XL10 := XL0old\cdot\left(\frac{VL1old}{Vb2}\right)^2\cdot\left(\frac{Sb}{SLine}\right) \qquad XL10 = 0.9\,pu$$

Transformer 2:

$$XT2old := 0.08pu$$

$$VT2old := 120kV \qquad ST2 := 100MVA$$

$$XT21 := XT2old\cdot\left(\frac{VT2old}{Vb2}\right)^2\cdot\left(\frac{Sb}{ST2}\right) \qquad XT21 = 0.0871\,pu$$

$$XT22 := XT21 \qquad XT22 = 0.0871\,pu$$

$$XT20 := XT21 \qquad XT20 = 0.0871\,pu$$

Generator 2:

$$Sg2 := 100MVA \qquad XG21old := 0.2pu$$

$$Vg2 := 13.2kV$$

$$XG21 := XG21old\cdot\left(\frac{Vg2}{Vb3}\right)^2\cdot\left(\frac{Sb}{Sg2}\right) \qquad XG21 = 0.2178\,pu$$

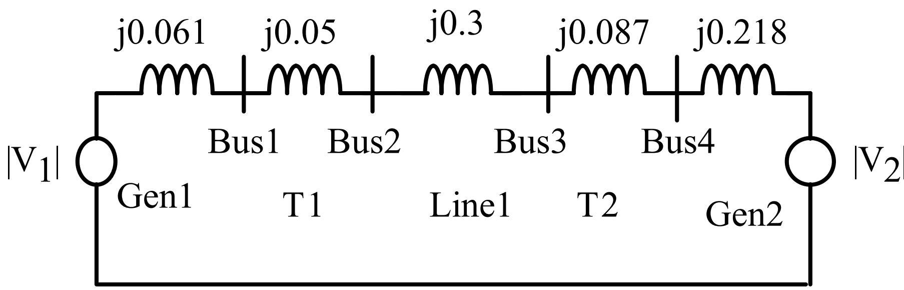
$$XG22 := XG21 \qquad XG22 = 0.2178\,pu$$

$$ZG20old := 10.0 + j\cdot 0.05pu$$

$$ZG20 := ZG20old\cdot\left(\frac{Vg2}{Vb3}\right)^2\cdot\left(\frac{Sb}{Sg2}\right)$$

$$ZG20 = (10.8885 + 0.0544i)\,pu$$

Positive Sequence Network (unreduced):



- Calculate reduced equivalent for each fault location

Parallel impedance branches: general equation as a function of location on line 1:

$$Z1equiv(M) := \left[\frac{1}{j\cdot XG11 + j\cdot XT11 + j\cdot XL11\cdot M} + \frac{1}{j\cdot XL11\cdot(1-M) + j\cdot XT21 + j\cdot XG21}\right]^{-1}$$

Fault at Bus 2:

$$Z1equiv2 := Z1equiv(0) \qquad Z1equiv2 = 0.0938i\,pu$$

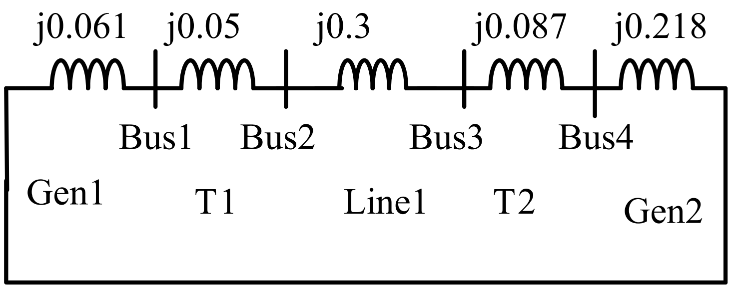
Fault at Bus 3:

$$Z1equiv3 := Z1equiv(1) \qquad Z1equiv3 = 0.175i\,pu$$

Fault at Bus M (midpoint of line):

$$Z1equivM := Z1equiv(0.5) \qquad Z1equivM = 0.1658i\,pu$$

Negative Sequence Network (unreduced):



- Calculate reduced equivalent for each fault location

Parallel impedance branches: general equation as a function of location on line 1

$$Z2equiv(M) := \left[\frac{1}{j \cdot XG12 + j \cdot XT12 + j \cdot XL12 \cdot M} + \frac{1}{j \cdot XL12 \cdot (1 - M) + j \cdot XT22 + j \cdot XG22}\right]^{-1}$$

Fault at Bus 2:

$$Z2equiv2 := Z2equiv(0) \quad Z2equiv2 = 0.0938i\,pu$$

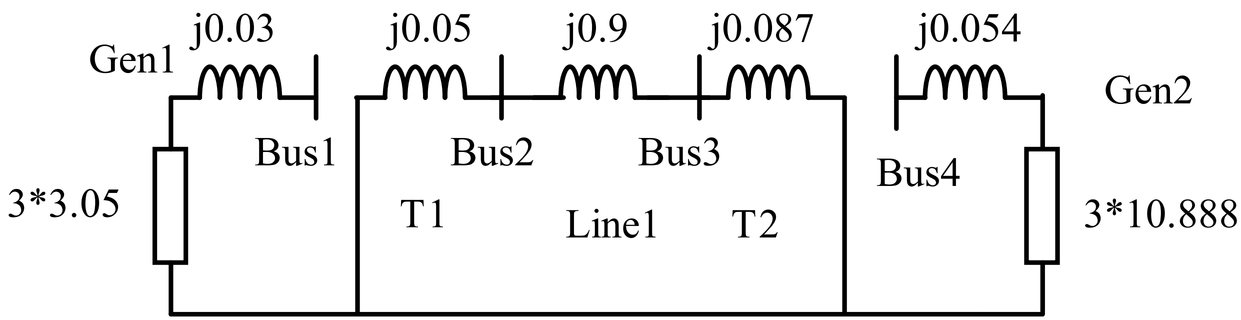
Fault at Bus 3:

$$Z2equiv3 := Z2equiv(1) \quad Z2equiv3 = 0.175i\,pu$$

Fault at Bus M (midpoint of line):

$$Z2equivM := Z2equiv(0.5) \quad Z2equivM = 0.1658i\,pu$$

Zero Sequence Network (unreduced):



- Calculate reduced equivalent for each fault location

Parallel impedance branches: general equation as a function of location on line 1

$$Z0equiv(M) := \left[\frac{1}{j\cdot XT10 + j\cdot XL10\cdot M} + \frac{1}{j\cdot XL10\cdot(1-M) + j\cdot XT20}\right]^{-1}$$

Fault at Bus 2:

$Z0equiv2 := Z0equiv(0)$ $Z0equiv2 = 0.0476i\,pu$

Fault at Bus 3:

$Z0equiv3 := Z0equiv(1)$ $Z0equiv3 = 0.0798i\,pu$

Fault at Bus M (midpoint of line):

$Z0equivM := Z0equiv(0.5)$ $Z0equivM = 0.2589i\,pu$

Create Ybus Matrices for Positive, Negative and Zero Sequences

$$Y1 := \begin{pmatrix} \frac{1}{jXG11} + \frac{1}{jXT11} & \frac{-1}{jXT11} & 0 & 0 \\ \frac{-1}{jXT11} & \frac{1}{jXT11} + \frac{1}{j \cdot XL11} & \frac{-1}{j \cdot XL11} & 0 \\ 0 & \frac{-1}{j \cdot XL11} & \frac{1}{jXT21} + \frac{1}{j \cdot XL11} & \frac{-1}{jXT21} \\ 0 & 0 & \frac{-1}{jXT21} & \frac{1}{jXT21} + \frac{1}{jXG21} \end{pmatrix}$$

$$Z\_1 := Y1^{-1}$$

$$Z\_1 = \begin{pmatrix} 0.0558i & 0.0515i & 0.026i & 0.0186i \\ 0.0515i & 0.0938i & 0.0473i & 0.0338i \\ 0.026i & 0.0473i & 0.175i & 0.125i \\ 0.0186i & 0.0338i & 0.125i & 0.1515i \end{pmatrix}$$

Note that Z_1 is no longer a sparse matrix.....

Negative sequence:

$$Y2 := \begin{pmatrix} \frac{1}{jXG12} + \frac{1}{jXT12} & \frac{-1}{jXT12} & 0 & 0 \\ \frac{-1}{jXT12} & \frac{1}{jXT12} + \frac{1}{j \cdot XL12} & \frac{-1}{j \cdot XL12} & 0 \\ 0 & \frac{-1}{j \cdot XL12} & \frac{1}{jXT22} + \frac{1}{j \cdot XL12} & \frac{-1}{jXT22} \\ 0 & 0 & \frac{-1}{jXT22} & \frac{1}{jXT22} + \frac{1}{jXG22} \end{pmatrix}$$

$$Z\_2 := Y2^{-1}$$

$$Z\_2 = \begin{pmatrix} 0.0558i & 0.0515i & 0.026i & 0.0186i \\ 0.0515i & 0.0938i & 0.0473i & 0.0338i \\ 0.026i & 0.0473i & 0.175i & 0.125i \\ 0.0186i & 0.0338i & 0.125i & 0.1515i \end{pmatrix}$$

The zero sequence matrix will be different, since there are more open connections and shorts to ground.

$$Y0 := \begin{pmatrix} \frac{1}{ZG10} & 0 & 0 & 0 \\ 0 & \frac{1}{jXT10} + \frac{1}{j \cdot XL10} & \frac{-1}{j \cdot XL10} & 0 \\ 0 & \frac{-1}{j \cdot XL10} & \frac{1}{jXT20} + \frac{1}{j \cdot XL10} & 0 \\ 0 & 0 & 0 & \frac{1}{ZG20} \end{pmatrix}$$

$$Z\_0 := Y0^{-1}$$

$$Z\_0 = \begin{pmatrix} 3.0498 + 0.0305i & 0 & 0 & 0 \\ 0 & 0.0476i & 0.0042i & 0 \\ 0 & 0.0042i & 0.0798i & 0 \\ 0 & 0 & 0 & 10.8885 + 0.0544i \end{pmatrix}$$

So for a SLG fault at bus 2, with prefault voltage of 1.0pu:

$Zf0 := Z\_0_{1,1}$ $\quad Zf0 = 0.0476i$

$Zf1 := Z\_1_{1,1}$ $\quad Zf1 = 0.0938i$

$Zf2 := Z\_2_{1,1}$ $\quad Zf2 = 0.0938i$

For comparison, if we had done the normal circuit reduction for this fault point we would have:

$Z1equiv2 = 0.0938i$ $\quad Z2equiv2 = 0.0938i$ $\quad Z0equiv2 = 0.0476i$

And for a SLG fault at bus 3, with prefault voltage of 1.0pu:

$Zf03 := Z\_0_{2,2}$ $\quad Zf03 = 0.0798i$

$Zf13 := Z\_1_{2,2}$ $\quad Zf13 = 0.175i$

$Zf23 := Z\_2_{2,2}$ $\quad Zf23 = 0.175i$

For comparison, if we had done the normal circuit reduction for this fault point we would have:

$Z1equiv3 = 0.175i$ $\quad Z2equiv3 = 0.175i$ $\quad Z0equiv3 = 0.0798i$

So we're consistent

SLG current for a fault at bus 2:

$$Iaf0 := \frac{1.0}{Z\_0_{1,1} + Z\_1_{1,1} + Z\_2_{1,1}} \qquad Iaf0 = -4.2524i \qquad Iaf1 := Iaf0$$

$$Iaf2 := Iaf0$$

Using plain circuit analysis we had:

$$I0\_f2 := \frac{1.0}{Z0equiv2 + Z1equiv2 + Z2equiv2} \qquad I0\_f2 = -4.2524i\,pu$$

So we would see:

$$\begin{pmatrix} Iafn \\ Ibfn \\ Icfn \end{pmatrix} := A1 \cdot \begin{pmatrix} Iaf0 \\ Iaf0 \\ Iaf0 \end{pmatrix} \qquad Iafn = -12.7572i \qquad Ibfn = 1.3 \times 10^{-15}$$

$$Icfn = 1.3 \times 10^{-15}$$

$$\begin{pmatrix} Vfn0 \\ Vfn1 \\ Vfn2 \end{pmatrix} := \begin{pmatrix} 0 \\ 1.0 \\ 0 \end{pmatrix} - \begin{pmatrix} Z\_0_{1,1} & 0 & 0 \\ 0 & Z\_1_{1,1} & 0 \\ 0 & 0 & Z\_2_{1,1} \end{pmatrix} \cdot \begin{pmatrix} Iaf0 \\ Iaf0 \\ Iaf0 \end{pmatrix}$$

$$\begin{pmatrix} Vafn \\ Vbfn \\ Vcfn \end{pmatrix} := A1 \cdot \begin{pmatrix} Vfn0 \\ Vfn1 \\ Vfn2 \end{pmatrix} \qquad \begin{pmatrix} Vafn \\ Vbfn \\ Vcfn \end{pmatrix} = \begin{pmatrix} 0 \\ -0.3036 - 0.866i \\ -0.3036 + 0.866i \end{pmatrix} \qquad \begin{pmatrix} |Vafn| \\ |Vbfn| \\ |Vcfn| \end{pmatrix} = \begin{pmatrix} 0 \\ 0.9177 \\ 0.9177 \end{pmatrix}$$

Bus voltages:

To determine the change in Positive Sequence voltages multiply Z1 by a vector of all zeros, with -1 times the positive sequence fault current injected at the faulted bus. The result is the ***CHANGE*** in positive sequence voltage at each bus due to the fault at bus 2.

$$\Delta V1 := Z\_1 \cdot \begin{pmatrix} 0 \\ -Iaf1 \\ 0 \\ 0 \end{pmatrix} \qquad \Delta V1 = \begin{pmatrix} -0.2192 \\ -0.3988 \\ -0.201 \\ -0.1436 \end{pmatrix}$$

The positive sequence voltages at each bus are then (note instead of 1+j0 we can also use power flow results):

$$V1 := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \Delta V1 \qquad V1 = \begin{pmatrix} 0.7808 \\ 0.6012 \\ 0.799 \\ 0.8564 \end{pmatrix}$$

Change in Negative Sequence voltages:

$$\Delta V2 := Z\_2 \cdot \begin{pmatrix} 0 \\ -Iaf2 \\ 0 \\ 0 \end{pmatrix} \qquad \Delta V2 = \begin{pmatrix} -0.2192 \\ -0.3988 \\ -0.201 \\ -0.1436 \end{pmatrix}$$

Negative sequence voltages

$$V2 := \Delta V2 \qquad V2 = \begin{pmatrix} -0.2192 \\ -0.3988 \\ -0.201 \\ -0.1436 \end{pmatrix}$$

Change in Zero Sequence voltages:

$$\Delta V0 := Z\_0 \cdot \begin{pmatrix} 0 \\ -Iaf0 \\ 0 \\ 0 \end{pmatrix} \qquad \Delta V0 = \begin{pmatrix} 0 \\ -0.2024 \\ -0.0179 \\ 0 \end{pmatrix}$$

Zero sequence voltages

$$V0 := \Delta V0 \qquad V0 = \begin{pmatrix} 0 \\ -0.2024 \\ -0.0179 \\ 0 \end{pmatrix}$$

Then we create the ABC domain voltages at any bus. For example, at Bus 2 we see:

$$Vabc\_f := A1 \cdot \begin{pmatrix} V0_{1,0} \\ V1_{1,0} \\ V2_{1,0} \end{pmatrix} \qquad Vabc\_f = \begin{pmatrix} 0 \\ -0.3036 - 0.866i \\ -0.3036 + 0.866i \end{pmatrix} \qquad \overrightarrow{|Vabc\_f|} = \begin{pmatrix} 0 \\ 0.9177 \\ 0.9177 \end{pmatrix}$$

This is the same as we found above using brute force methods

Fault current contributions:

From bus 1: Current on the Y connected side of the transformer where there is a zero sequence path.

$$I1bus1 := \frac{V1_{0,0} - V1_{1,0}}{jXT11} \qquad I1bus1 = -3.5931i$$

$$I2bus1 := \frac{V2_{0,0} - V2_{1,0}}{jXT12} \qquad I2bus1 = -3.5931i$$

$$I0bus1 := \frac{0 - V0_{1,0}}{jXT10} \qquad I0bus1 = -4.0474i$$

This is what was calculated using the methods for the exam.

Note that the zero sequence term has (0 -V) since one end of inductance is grounded.

Currents on the Delta Side:

However we also need to include the phase shift in the symmetrical components as we cross the transformer to get the currents on the Δ side.

Assuming an ANSI standard connection, then the phase voltage on the Y side will lead the phase voltage on the delta side by 30 deg in the positive sequence. The currents will see the same shift.

In addition, the negative sequence voltage sees the opposite shift as the positive sequence as do the currents. So:

$I0gen := 0$

$I1gen := I1bus1 \cdot e^{-j \cdot 30deg}$ $\qquad |I1gen| = 3.5931$ $\qquad arg(I1gen) = -120\,deg$

$I2gen := I2bus1 \cdot e^{j \cdot 30deg}$ $\qquad |I2gen| = 3.5931$ $\qquad arg(I2gen) = -60\,deg$

$$\begin{pmatrix} Iag1 \\ Ibg1 \\ Icg1 \end{pmatrix} := A1 \cdot \begin{pmatrix} I0gen \\ I1gen \\ I2gen \end{pmatrix}$$

$Iag1 = -6.2234i$ $\qquad |Iag1| = 6.2234$ $\qquad arg(Iag1) = -90\,deg$

$Ibg1 = 6.2234i$ $\qquad |Ibg1| = 6.2234$ $\qquad arg(Ibg1) = 90\,deg$ $\qquad$ so it looks like a L-L fault.

$Icg1 = 0$ $\qquad |Icg1| = 1.1102 \times 10^{-15}$ $\qquad (Icg1) = 126.8699\,deg$

Current through transformer 2 due to fault at bus 2 (seen from Y side):

$$I1T2 := \frac{V1_{3,0} - V1_{2,0}}{j \cdot XT21} \qquad I1T2 = -0.6593i$$

$$I2T2 := \frac{V2_{3,0} - V2_{2,0}}{j \cdot XT22} \qquad I2T2 = -0.6593i$$

Again, these match the results from earlier.

$$I0T2 := \frac{0 - V0_{2,0}}{j \cdot XT20} \qquad I0T2 = -0.205i$$

$$IabcT2 := A1 \cdot \begin{pmatrix} I0T2 \\ I1T2 \\ I2T2 \end{pmatrix} \qquad \overrightarrow{|IabcT2|} = \begin{pmatrix} 1.5237 \\ 0.4543 \\ 0.4543 \end{pmatrix}$$

$$\overrightarrow{\arg(IabcT2)} = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} deg$$

Alternate Approach for Getting Relevant Parts of Zbus:

Rather than inverting the Sparse Ybus, it is more efficient to take the LU factored Ybus, and just compute the part of the Zbus that is actually needed.

In the example above, we only needed the data related to bus 2:

$M0 := \text{lu}(Y0)$

$P0 := \text{submatrix}(M0, 0, 3, 0, 3)$

$L0 := \text{submatrix}(M0, 0, 3, 4, 7)$ lower triangular

$U0 := \text{submatrix}(M0, 0, 3, 8, 11)$ upper triangular

Solve the following two steps:

$$L0 \cdot Y0 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

The column vector has a 1 for the faulted bus, and zero's everywhere else.

Solve for Y0 using forward substitution

and then: $U0 \cdot ZbusColumn2 = Y0$

Solve for ZbusColumn2 using back substitution

Normally this next step is done as a forward substitution for a large system, but I'm cheating since this is small. Note that the permutation matrix is used to restore the correct row and column ordering.

$$Y0bus2 := L0^{-1} \cdot P0 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad Y0bus2 = \begin{pmatrix} 0 \\ 1 \\ 0.0526 \\ 0 \end{pmatrix}$$

$$Zf2\_col2\_0 := U0^{-1} \cdot Y0bus2$$

$$Zf2\_col2\_0 = \begin{pmatrix} 0 \\ 0.0476i \\ 0.0042i \\ 0 \end{pmatrix}$$

The transpose of this is row 2

Compare to the original Zbus matrix (this result matches column 2):

$$Z\_0 = \begin{pmatrix} 3.0498 + 0.0305i & 0 & 0 & 0 \\ 0 & 0.0476i & 0.0042i & 0 \\ 0 & 0.0042i & 0.0798i & 0 \\ 0 & 0 & 0 & 10.8885 + 0.0544i \end{pmatrix}$$

This vector along with the positive and negative sequence values can then be used to analyze the fault. This is what most commercial fault programs do.

Transformer Phase Shift:

Now add the effect of the 30 degree phase shift of the transformer

$$Y1a := \begin{pmatrix} \frac{1}{jXG11} + \frac{1}{jXT11} & \frac{-1 \cdot e^{-j \cdot 30deg}}{jXT11} & 0 & 0 \\ \frac{-1 \cdot e^{j \cdot 30deg}}{jXT11} & \frac{1}{jXT11} + \frac{1}{j \cdot XL11} & \frac{-1}{j \cdot XL11} & 0 \\ 0 & \frac{-1}{j \cdot XL11} & \frac{1}{jXT21} + \frac{1}{j \cdot XL11} & \frac{-1 \cdot e^{j \cdot 30deg}}{jXT21} \\ 0 & 0 & \frac{-1 \cdot e^{-j \cdot 30deg}}{jXT21} & \frac{1}{jXT21} + \frac{1}{jXG21} \end{pmatrix}$$

Note that the phase shifts for the two transformers are opposite, since one is a step up in voltage and the other is a step down.

$$Y1a = \begin{pmatrix} -36.3946i & 10 + 17.3205i & 0 & 0 \\ -10 + 17.3205i & -23.3333i & 3.3333i & 0 \\ 0 & 3.3333i & -14.8134i & -5.74 + 9.942i \\ 0 & 0 & 5.74 + 9.942i & -16.072i \end{pmatrix}$$

Opposite phase shift in the negative sequence matrix

$$Y2a := \begin{pmatrix} \frac{1}{jXG11} + \frac{1}{jXT11} & \frac{-1 \cdot e^{j \cdot 30deg}}{jXT11} & 0 & 0 \\ \frac{-1 \cdot e^{-j \cdot 30deg}}{jXT11} & \frac{1}{jXT11} + \frac{1}{j \cdot XL11} & \frac{-1}{j \cdot XL11} & 0 \\ 0 & \frac{-1}{j \cdot XL11} & \frac{1}{jXT21} + \frac{1}{j \cdot XL11} & \frac{-1 \cdot e^{-j \cdot 30deg}}{jXT21} \\ 0 & 0 & \frac{-1 \cdot e^{j \cdot 30deg}}{jXT21} & \frac{1}{jXT21} + \frac{1}{jXG21} \end{pmatrix}$$

$$Y2a = \begin{pmatrix} -36.3946i & -10 + 17.3205i & 0 & 0 \\ 10 + 17.3205i & -23.3333i & 3.3333i & 0 \\ 0 & 3.3333i & -14.8134i & 5.74 + 9.942i \\ 0 & 0 & -5.74 + 9.942i & -16.072i \end{pmatrix}$$

No shift in the zero sequence, so we don't need to add it at all and can retain what we had .

$M1 := lu(Y1a)$

$P1 := submatrix(M1, 0, 3, 0, 3)$

$L1 := submatrix(M1, 0, 3, 4, 7)$ lower triangular

$U1 := submatrix(M1, 0, 3, 8, 11)$ upper triangular

$$Y1bus2 := L1^{-1} \cdot P1 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Y1bus2 = \begin{pmatrix} 0 \\ 1 \\ 0.2701 \\ 0.193 - 0.1114i \end{pmatrix}$$

$Zf2\_col2\_1 := U1^{-1} \cdot Y1bus2$

$$Zf2\_col2\_1 = \begin{pmatrix} 0.0258 + 0.0446i \\ 0.0938i \\ 0.0473i \\ 0.0169 + 0.0292i \end{pmatrix}$$

$M2 := lu(Y2a)$

$P2 := submatrix(M2, 0, 3, 0, 3)$

$L2 := submatrix(M2, 0, 3, 4, 7)$ lower triangular

$U2 := submatrix(M2, 0, 3, 8, 11)$ upper triangular

$$Y2bus2 := L2^{-1} P2 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Y2bus2 = \begin{pmatrix} 0 \\ 1 \\ 0.2701 \\ 0.193 + 0.1114i \end{pmatrix}$$

$Zf2\_col2\_2 := U2^{-1} \cdot Y2bus2$

$$Zf2\_col2\_2 = \begin{pmatrix} -0.0258 + 0.0446i \\ 0.0938i \\ 0.0473i \\ -0.0169 + 0.0292i \end{pmatrix}$$

S for a SLG fault at bus 2, with prefault voltage of 1.0pu: $Zf1 = 0.0938i$

$Zf0 := Zf2\_col2\_0_{1,0}$ $Zf0 = 0.0476i$

$Zf1 := Zf2\_col2\_1_{1,0}$ $Zf1 = 0.0938i$

$Zf2 := Zf2\_col2\_2_{1,0}$ $Zf2 = 0.0938i$

Still the same as above, phase shift doesn't impact this part.

$Z1equiv2 = 0.0938i$ $Z2equiv2 = 0.0938i$ $Z0equiv2 = 0.0476i$

$$Ialtf0 := \frac{1.0}{Z\_0_{1,1} + Z\_1_{1,1} + Z\_2_{1,1}}$$

$Ialtf0 = -4.2524i$ $Ialtf1 := Ialtf0$

$Ialtf2 := Ialtf0$

This is the same as we found earlier.

Bus voltages:

Change in Positive Sequence voltages:

Positive sequence voltages $\Delta V1alt := Zf2\_col2\_1 \cdot (-Ialtf1)$

$$\Delta V1alt = \begin{pmatrix} -0.1898 + 0.1096i \\ -0.3988 \\ -0.201 \\ -0.1243 + 0.0718i \end{pmatrix}$$

$$V1alt := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \Delta V1alt \qquad V1alt = \begin{pmatrix} 0.8102 + 0.1096i \\ 0.6012 \\ 0.799 \\ 0.8757 + 0.0718i \end{pmatrix}$$

note that without the phase shifts:

$$V1 = \begin{pmatrix} 0.7808 \\ 0.6012 \\ 0.799 \\ 0.8564 \end{pmatrix}$$

Change in Negative Sequence voltages:

$$\Delta V2alt := Zf2\_col2\_2 \cdot (-Ialtf2) \qquad \Delta V2alt = \begin{pmatrix} -0.1898 - 0.1096i \\ -0.3988 \\ -0.201 \\ -0.1243 - 0.0718i \end{pmatrix}$$

Note different signs on imaginary parts crossing the transformer.

Negative sequence voltages

$$V2alt := \Delta V2alt \qquad V2alt = \begin{pmatrix} -0.1898 - 0.1096i \\ -0.3988 \\ -0.201 \\ -0.1243 - 0.0718i \end{pmatrix}$$

Change in Zero Sequence voltages:

$$\Delta V0alt := Zf2\_col2\_0 \cdot (-Ialtf0) \qquad \Delta V0alt = \begin{pmatrix} 0 \\ -0.2024 \\ -0.0179 \\ 0 \end{pmatrix}$$

originally:

$$\Delta V0 = \begin{pmatrix} 0 \\ -0.2024 \\ -0.0179 \\ 0 \end{pmatrix}$$

No change, since no phase shift in zero sequence

Zero sequence voltages

$$V0alt := \Delta V0alt \qquad V0alt = \begin{pmatrix} 0 \\ -0.2024 \\ -0.0179 \\ 0 \end{pmatrix}$$

As a check:

$$Vabc\_falt := A1 \cdot \begin{pmatrix} V0alt_{1,0} \\ V1alt_{1,0} \\ V2alt_{1,0} \end{pmatrix} \qquad Vabc\_falt = \begin{pmatrix} 0 \\ -0.3036 - 0.866i \\ -0.3036 + 0.866i \end{pmatrix}$$

Same as we found without the shift....

Fault current contributions:

From bus 1: (with the phase shift)

$$I1bus1 := \frac{V1_{0,0} - V1_{1,0}}{\frac{-1}{Y1a_{0,1}}} \qquad I1bus1 = -1.7965 - 3.1117i$$

Divide by the modified Ybus terms....

$$I2bus1 := \frac{V2_{0,0} - V2_{1,0}}{\frac{-1}{Y2a_{0,1}}} \qquad I2bus1 = 1.7965 - 3.1117i$$

$I0bus1 := 0$ open circuit

$$Igenalt := A1 \cdot \begin{pmatrix} I0bus1 \\ I1bus1 \\ I2bus1 \end{pmatrix} \quad Igenalt = \begin{pmatrix} -6.2234i \\ 6.2234i \\ 1.7764i \times 10^{-15} \end{pmatrix} \quad \overrightarrow{|Igenalt|} = \begin{pmatrix} 6.2234 \\ 6.2234 \\ 1.8971 \times 10^{-15} \end{pmatrix} \quad \overrightarrow{arg(Igenalt)} = \begin{pmatrix} -90 \\ 90 \\ 110.556 \end{pmatrix} deg$$

Where, the earlier calculations had:

$|Iag1| = 6.2234$ $\qquad arg(Iag1) = -90\,deg$

$|Ibg1| = 6.2234$ $\qquad arg(Ibg1) = 90\,deg$

$|Icg1| = 1.1102 \times 10^{-15}$ $\qquad arg(Icg1) = 126.8699\,deg$

Which is the same as above other than rounding errors on the phase C term....

Mid-line Faults

Now suppose we have a fault 50% of the way down the line between buses 2 and 3. We can modify the matrix by adding a new bus at the fault location. We need to create a new Ybus (there are compensation based methods to avoid this, but we won't get into those here. I'm using the Ybus without the phase shifts for now.

Set fault location: $m := 0.5$

$$Y1m := \begin{bmatrix} \frac{1}{jXG11}+\frac{1}{jXT11} & \frac{-1}{jXT11} & 0 & 0 & 0 \\ \frac{-1}{jXT11} & \frac{1}{jXT11}+\frac{1}{j\cdot(m\cdot XL11)} & 0 & 0 & \frac{-1}{j\cdot(m\cdot XL11)} \\ 0 & 0 & \frac{1}{jXT21}+\frac{1}{j\cdot(1-m)\cdot XL11} & \frac{-1}{jXT21} & \frac{-1}{j\cdot(1-m)\cdot XL11} \\ 0 & 0 & \frac{-1}{jXT21} & \frac{1}{jXT21}+\frac{1}{jXG21} & 0 \\ 0 & \frac{-1}{j\cdot(m\cdot XL11)} & \frac{-1}{j\cdot(1-m)\cdot XL11} & 0 & \frac{1}{j\cdot(m\cdot XL11)}+\frac{1}{j\cdot(1-m)\cdot XL11} \end{bmatrix}$$

- The extra bus as created by adding an extra row and column to the matrix. The old entry connecting bus 2 to bus 3 is zeroed out.
- Also note that the entries on the diagonal where the line appears are modified.

$$Y2m := \begin{bmatrix} \frac{1}{jXG12}+\frac{1}{jXT12} & \frac{-1}{jXT12} & 0 & 0 & 0 \\ \frac{-1}{jXT12} & \frac{1}{jXT12}+\frac{1}{j\cdot(m\cdot XL12)} & 0 & 0 & \frac{-1}{j\cdot(m\cdot XL12)} \\ 0 & 0 & \frac{1}{jXT22}+\frac{1}{j\cdot(1-m)\cdot XL12} & \frac{-1}{jXT22} & \frac{-1}{j\cdot(1-m)\cdot XL12} \\ 0 & 0 & \frac{-1}{jXT22} & \frac{1}{jXT22}+\frac{1}{jXG22} & 0 \\ 0 & \frac{-1}{j\cdot(m\cdot XL12)} & \frac{-1}{j\cdot(1-m)\cdot XL12} & 0 & \frac{1}{j\cdot(m\cdot XL12)}+\frac{1}{j\cdot(1-m)\cdot XL12} \end{bmatrix}$$

$$Y0m := \begin{bmatrix} \frac{1}{ZG10} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{jXT10}+\frac{1}{j\cdot(m\cdot XL10)} & 0 & 0 & \frac{-1}{j\cdot(m\cdot XL10)} \\ 0 & 0 & \frac{1}{jXT20}+\frac{1}{j\cdot(1-m)\cdot XL10} & 0 & \frac{-1}{j\cdot(1-m)\cdot XL10} \\ 0 & 0 & 0 & \frac{1}{ZG20} & 0 \\ 0 & \frac{-1}{j\cdot(m\cdot XL10)} & \frac{-1}{j\cdot(1-m)\cdot XL10} & 0 & \frac{1}{j\cdot(m\cdot XL10)}+\frac{1}{j\cdot(1-m)\cdot XL10} \end{bmatrix}$$

$Z1m := Y1m^{-1}$ $Z2m := Y2m^{-1}$ $Z0m := Y0m^{-1}$

So for a SLG fault at bus m we have the following impedances (note we need node 4,4 for the new bus.

$Zf0m := Z0m_{4,4}$ $Zf0m = 0.2589i$

$Zf1m := Z1m_{4,4}$ $Zf1m = 0.1658i$

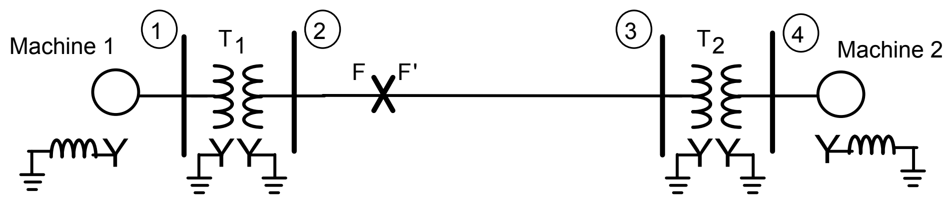
$Zf2m := Z2m_{4,4}$ $Zf2m = 0.1658i$

For comparison, if we had done the normal circuit reduction for this fault point
we would have:

$Z1equivM = 0.1658i$ $Z2equivM = 0.1658i$ $Z0equivM = 0.2589i$

In the system below, consider that machine 2 is a motor drawing a load equivalent to 80MVA at 0.85 power factor lagging and nominal system voltage of 345kV at bus 3. Determine the change in voltage at bus 3 when the transmission line undergoes (a) a one-open conductor fault and (b) a two-open conductor fault along its span between buses 2 and 3. Choose a base of 100MVA, 345kV in the transmission line.

- System one-line diagram:



Machines 1 and 2 $S_{Mach} := 100MVA$ $\quad V_{machine} := 20kV$

$X_{dMach''} := 20\%$ $\quad X_{1Mach} := X_{dMach''}$ $\quad X_{2Mach} := X_{1Mach}$

$X_{0Mach} := 4\%$ $\quad X_{nMach} := 5\%$

Transformers T1 and T2: $S_{Tran} := 1000MVA$ $\quad V_{HV} := 345kV$ $\quad V_{LV} := 20kV$ $\quad X_T := 8\%$

Transmission Line $\quad X_{L1} := 15\%$ $\quad X_{L2} := X_{L1}$ $\quad X_{L0} := 50\%$

$S_{Base} := 100MVA$

$V_{BLine} := 345kV$ $\quad V_{B\_mach} := V_{BLine} \cdot \left(\frac{V_{LV}}{V_{HV}}\right)$ $\quad V_{B\_mach} = 20\,kV$

No change of base calculations are needed for this system.

Determine internal source voltages:

$magS_{pre} := 80MVA \qquad pf_{pre} := 0.8 \quad \text{lagging} \qquad \theta_{pre} := acos(0.85) \qquad \theta_{pre} = 31.7883\,deg$

$$S_{pre} := \frac{magS_{pre}}{S_{Base}} \cdot e^{j\cdot\theta_{pre}} \qquad S_{pre} = (0.68 + 0.4214i)\,pu \quad \left|S_{pre}\right| = 0.8\,pu$$

Assume bus 3 voltage is 1.0 pu at and angle of 0 degrees.

$V_3 := 1.0$

$$I_{load} := \overline{\left(\frac{S_{pre}}{V_3}\right)} \qquad I_{load} = 0.68 - 0.4214i \qquad \left|I_{load}\right| = 0.8\,pu \qquad arg\left(I_{load}\right) = -31.7883\,deg$$

Internal voltage on the motor (since we don't know steady-state synchronous reactance, use X1):

$$E_2 := V_3 - I_{load}\cdot j\left(X_T + X_{1Mach}\right) \qquad \left|E_2\right| = 0.9023 \quad \phi_2 := arg\left(E_2\right) \qquad \phi_2 = -12.1817\,deg$$

Generator internal voltage:

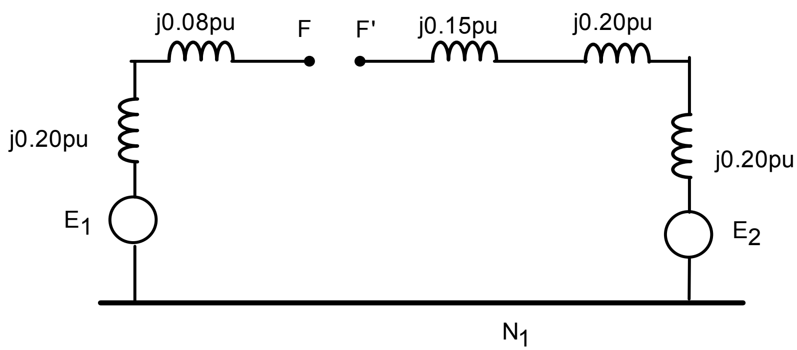
$$E_1 := V_3 + I_{load}\cdot\left(j\cdot X_{L1} + j\cdot X_T + j\cdot X_{1Mach}\right) \qquad \left|E_1\right| = 1.2169\,pu \quad \phi_1 := arg\left(E_1\right) \qquad \phi_1 = 13.9036\,deg$$

Check result by calculating power transfer between sources and current:

$$P_{trans} := \frac{\left|E_1\right|\cdot\left|E_2\right|\cdot\sin\left(\phi_1 - \phi_2\right)}{2\cdot X_{1Mach} + 2\cdot X_T + X_{L1}} \qquad P_{trans} - Re\left(S_{pre}\right) = 0$$

$$I_{trans} := \frac{E_1 - E_2}{j\left(2\cdot X_{1Mach} + 2\cdot X_T + X_{L1}\right)} \qquad I_{trans} - I_{load} = 0$$

- Positive sequence equivalent circuit (with phase open point indicated).



Find total impedance counterclockwise around loop from F to F'

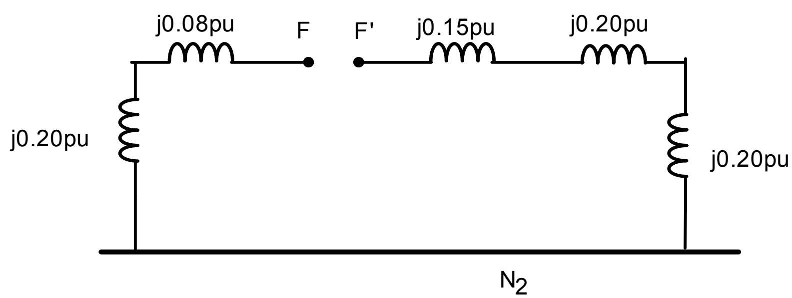
$$Z_{1total} := j\cdot\left(X_{1Mach} + X_T + X_{L1} + X_T + X_{1Mach}\right)$$

$$Z_{1total} = 0.71i\,pu$$

$$Z_{1FF'} := Z_{1total}$$

$$V_{equiv} := E_1 - E_2$$

- Negative sequence equivalent circuit:



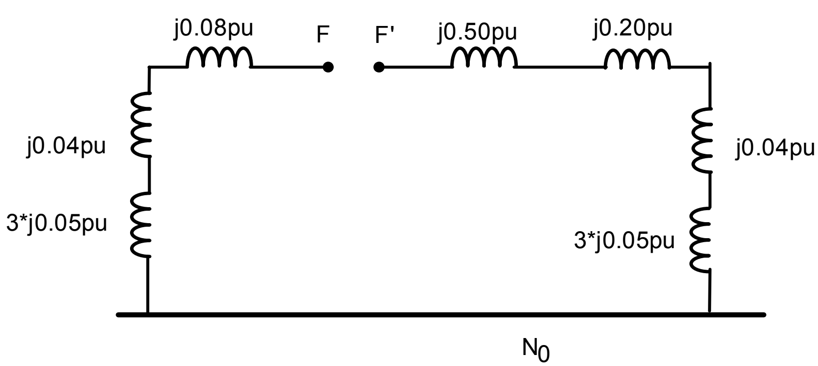
Find total impedance counterclockwise around loop from F to F'

$$Z_{2total} := j\cdot\left(X_{2Mach} + X_T + X_{L2} + X_T + X_{2Mach}\right)$$

$$Z_{2total} = 0.71i\,pu$$

$$Z_{2FF'} := Z_{2total}$$

- Zero sequence equivalent:



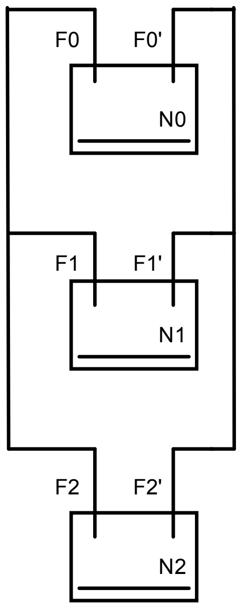
Find total impedance counterclockwise around loop from F to F'

$$Z_{0total} := j\cdot\left(2\cdot X_{0Mach} + 2\cdot X_T + X_{L0} + 2\cdot 3\cdot X_{nMach}\right)$$

$$Z_{0total} = 1.04i\,pu$$

$$Z_{0FF'} := Z_{0total}$$

Now solve for the single phase open circuit currents and voltages:



$$I_1 := \frac{V_{equiv}}{Z_{1FF'} + \left(\frac{1}{Z_{2FF'}} + \frac{1}{Z_{0FF'}}\right)^{-1}}$$

$I_1 = (0.4265 - 0.2643i)\,pu$

$|I_1| = 0.5018\,pu \qquad arg(I_1) = -31.7883\,deg$

$$I_2 := -I_1 \cdot \left(\frac{Z_{0FF'}}{Z_{2FF'} + Z_{0FF'}}\right)$$

$I_2 = (-0.2535 + 0.1571i)\,pu$

$|I_2| = 0.2982\,pu \qquad arg(I_2) = 148.2117\,deg$

$$I_0 := -I_1 \cdot \left(\frac{Z_{2FF'}}{Z_{2FF'} + Z_{0FF'}}\right)$$

$I_0 = (-0.173 + 0.1072i)\,pu$

$|I_0| = 0.2036\,pu \qquad arg(I_0) = 148.2117\,deg$

$$I_{abc} := A_{012} \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix} \qquad \overrightarrow{|I_{abc}|} = \begin{pmatrix} 0 \\ 0.7571 \\ 0.7571 \end{pmatrix} pu \qquad \overrightarrow{arg(I_{abc})} = \begin{pmatrix} 90 \\ -145.5749 \\ 81.9982 \end{pmatrix} deg$$

Using the right have the sequence equivalent circuits:

$V_{3new1} := E_2 + I_1 \cdot j \cdot (X_{1Mach} + X_T) \qquad |V_{3new1}| = 0.9586\,pu \qquad arg(V_{3new1}) = -4.2458\,deg$

$V_{3new2} := 0 + I_2 \cdot j \cdot (X_{2Mach} + X_T) \qquad |V_{3new2}| = 0.0835\,pu \qquad arg(V_{3new2}) = -121.7883\,deg$

$V_{3new0} := 0 + I_0 \cdot j \cdot (X_{0Mach} + X_T + 3 \cdot X_{nMach}) \qquad |V_{3new0}| = 0.055\,pu \qquad arg(V_{3new0}) = -121.7883\,deg$

$$V_{3newABC} := A_{012} \cdot \begin{pmatrix} V_{3new0} \\ V_{3new1} \\ V_{3new2} \end{pmatrix} \qquad \overrightarrow{\left|V_{3newABC}\right|} = \begin{pmatrix} 0.903 \\ 0.9715 \\ 1.0138 \end{pmatrix} pu \qquad \overrightarrow{\arg\left(V_{3newABC}\right)} = \begin{pmatrix} -12.06 \\ -119.9475 \\ 118.579 \end{pmatrix} deg$$

$$\Delta V_{ABC} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} - V_{3newABC} \qquad \overrightarrow{\left|\Delta V_{ABC}\right|} = \begin{pmatrix} 0.222 \\ 0.0285 \\ 0.0285 \end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V_{ABC}\right)} = \begin{pmatrix} 58.2117 \\ -121.7883 \\ -121.7883 \end{pmatrix} deg$$

Solve using the Zbus method. First get the Zbus matrices for the positive, negative and zero sequence networks:

$$Y_{bus1} := \begin{pmatrix} \frac{1}{j \cdot X_{1Mach}} + \frac{1}{j \cdot X_T} & \frac{-1}{j \cdot X_T} & 0 & 0 \\ \frac{-1}{j \cdot X_T} & \frac{1}{jX_T} + \frac{1}{jX_{L1}} & \frac{-1}{j \cdot X_{L1}} & 0 \\ 0 & \frac{-1}{j \cdot X_{L1}} & \frac{1}{jX_T} + \frac{1}{jX_{L1}} & \frac{-1}{j \cdot X_T} \\ 0 & 0 & \frac{-1}{j \cdot X_T} & \frac{1}{j \cdot X_{1Mach}} + \frac{1}{j \cdot X_T} \end{pmatrix} \qquad Z_{bus1} := Y_{bus1}^{-1}$$

$$Z_{bus1} = \begin{pmatrix} 0.1437i & 0.1211i & 0.0789i & 0.0563i \\ 0.1211i & 0.1696i & 0.1104i & 0.0789i \\ 0.0789i & 0.1104i & 0.1696i & 0.1211i \\ 0.0563i & 0.0789i & 0.1211i & 0.1437i \end{pmatrix} pu \qquad Z_{bus2} := Z_{bus1}$$

$$Y_{bus0} := \begin{pmatrix} \frac{1}{j \cdot X_{0Mach} + 3 \cdot j \cdot X_{nMach}} + \frac{1}{j \cdot X_T} & \frac{-1}{j \cdot X_T} & 0 & 0 \\ \frac{-1}{j \cdot X_T} & \frac{1}{jX_T} + \frac{1}{jX_{L0}} & \frac{-1}{j \cdot X_{L0}} & 0 \\ 0 & \frac{-1}{j \cdot X_{L0}} & \frac{1}{jX_T} + \frac{1}{jX_{L0}} & \frac{-1}{j \cdot X_T} \\ 0 & 0 & \frac{-1}{j \cdot X_T} & \frac{1}{j \cdot X_{0Mach} + 3 \cdot j \cdot X_{nMach}} + \frac{1}{j \cdot X_T} \end{pmatrix}$$

$$Z_{bus0} := Y_{bus0}^{-1}$$

$$Z_{bus0} = \begin{pmatrix} 0.1553i & 0.1407i & 0.0493i & 0.0347i \\ 0.1407i & 0.1999i & 0.0701i & 0.0493i \\ 0.0493i & 0.0701i & 0.1999i & 0.1407i \\ 0.0347i & 0.0493i & 0.1407i & 0.1553i \end{pmatrix} pu$$

Reset origin for matrices and vectors: $ORIGIN := 1$

Equivalent impedances looking into the network from the open segment:

$$Z_{1pp'} := \frac{-(j \cdot X_{L1})^2}{Z_{bus1_{2,2}} + Z_{bus1_{3,3}} - 2 \cdot Z_{bus1_{2,3}} - j \cdot X_{L1}}$$ $Z_{1pp'} = 0.71i\,pu$ Same as calculated above

$$Z_{2pp'} := Z_{1pp'}$$

$$Z_{0pp'} := \frac{-(j \cdot X_{L0})^2}{Z_{bus0_{2,2}} + Z_{bus0_{3,3}} - 2 \cdot Z_{bus0_{2,3}} - j \cdot X_{L0}} \qquad Z_{0pp'} = 1.04i\,\text{pu}$$

Same as calculated above

Find the sequence currents:

$$I_{1aopen} := I_{trans} \cdot \frac{Z_{1pp'}}{Z_{1pp'} + \left(\frac{1}{Z_{0pp'}} + \frac{1}{Z_{2pp'}}\right)^{-1}} \qquad |I_{1aopen}| = 0.5018\,\text{pu} \qquad \arg(I_{1aopen}) = -31.7883\,\text{deg}$$

same as above.

$$I_{2aopen} := -I_{1aopen} \cdot \left(\frac{Z_{0pp'}}{Z_{2pp'} + Z_{0pp'}}\right) \qquad |I_{2aopen}| = 0.2982\,\text{pu} \qquad \arg(I_{2aopen}) = 148.2117\,\text{deg}$$

$$I_{0aopen} := -I_{1aopen} \cdot \left(\frac{Z_{2pp'}}{Z_{2pp'} + Z_{0pp'}}\right) \qquad |I_{0aopen}| = 0.2036\,\text{pu} \qquad \arg(I_{0aopen}) = 148.2117\,\text{deg}$$

Sequence voltages across the open circuit

$$V_{1Aopen} := I_{1aopen} \cdot \frac{Z_{2pp'} \cdot Z_{0pp'}}{Z_{2pp'} + Z_{0pp'}} \qquad |V_{1Aopen}| = 0.2117\,\text{pu} \qquad \arg(V_{1Aopen}) = 58.2117\,\text{deg}$$

Since the positive, negative and zero sequence voltages are equal for the phase A open case:

$$V_{2Aopen} := V_{1Aopen}$$

$V_{0Aopen} := V_{1Aopen}$

Then the change in voltage at Bus 3 is:

$\Delta V_{3\_1aopen} := \left(\frac{Zbus1_{3,2} - Zbus1_{3,3}}{j \cdot X_{L1}}\right) \cdot V_{1Aopen}$ $\quad \left|\Delta V_{3\_1aopen}\right| = 0.0835\,pu$ $\quad \arg\left(\Delta V_{3\_1aopen}\right) = -121.7883\,deg$

$\Delta V_{3\_2aopen} := \left(\frac{Zbus2_{3,2} - Zbus2_{3,3}}{j \cdot X_{L2}}\right) \cdot V_{2Aopen}$ $\quad \left|\Delta V_{3\_2aopen}\right| = 0.0835\,pu$ $\quad \arg\left(\Delta V_{3\_2aopen}\right) = -121.7883\,deg$

$\Delta V_{3\_0aopen} := \left(\frac{Zbus0_{3,2} - Zbus0_{3,3}}{j \cdot X_{L0}}\right) \cdot V_{0Aopen}$ $\quad \left|\Delta V_{3\_0aopen}\right| = 0.055\,pu$ $\quad \arg\left(\Delta V_{3\_0aopen}\right) = -121.7883\,deg$

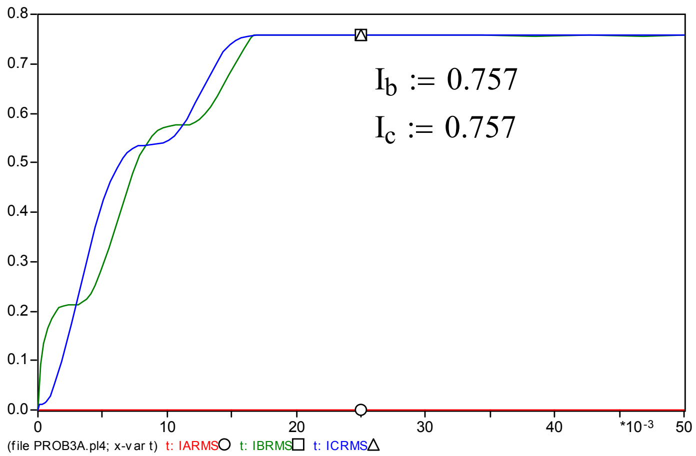
$\Delta V_{3ABC\_book} := A_{012} \cdot \begin{pmatrix} \Delta V_{3\_0aopen} \\ \Delta V_{3\_1aopen} \\ \Delta V_{3\_2aopen} \end{pmatrix}$ $\quad \overrightarrow{\left|\Delta V_{3ABC\_book}\right|} = \begin{pmatrix} 0.222 \\ 0.0285 \\ 0.0285 \end{pmatrix} pu$ $\quad \overrightarrow{\arg\left(\Delta V_{3ABC\_book}\right)} = \begin{pmatrix} -121.7883 \\ 58.2117 \\ 58.2117 \end{pmatrix} deg$

Then using the Zbus method, the voltage at bus 3 would be:

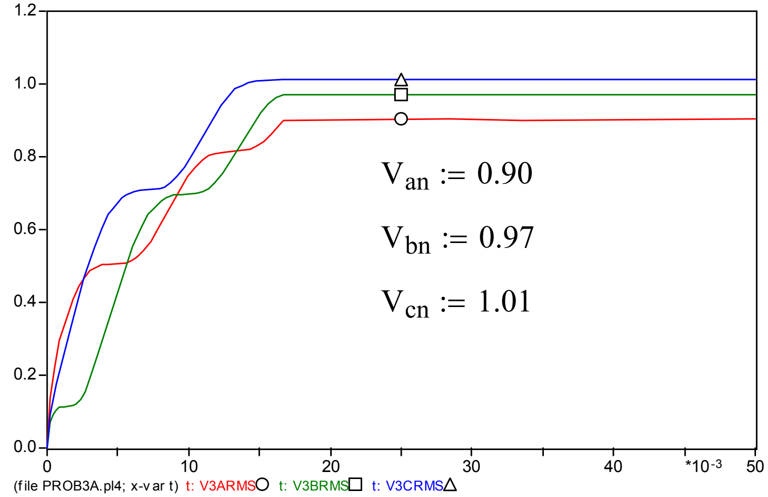
$V_{3ABC\_aopen} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} + \Delta V_{3ABC\_book}$ $\quad \overrightarrow{\left|V_{3ABC\_aopen}\right|} = \begin{pmatrix} 0.903 \\ 0.9715 \\ 1.0138 \end{pmatrix} pu$ $\quad \arg\left(V_{3ABC\_aopen}\right) = \begin{pmatrix} -12.06 \\ -119.9475 \\ 118.579 \end{pmatrix} deg$

Note the plus sign instead of the minus sign as in the circuit based case above.

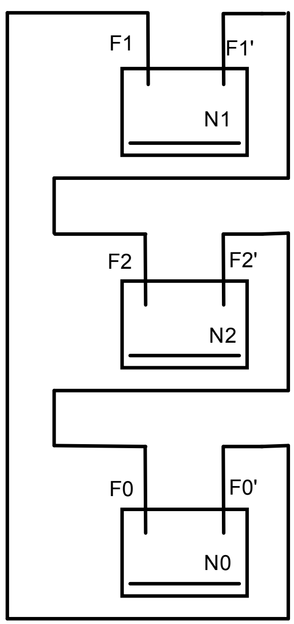
ATP simulation results: Currents



Voltages



Now solve the two phase open circuit below for the sequence currents:



$$I_1 := \frac{V_{equiv}}{Z_{1FF'} + Z_{2FF'} + Z_{0FF'}}$$

$$I_1 = (0.1963 - 0.1216i)\,\text{pu}$$

$$|I_1| = 0.2309\,\text{pu} \qquad \arg(I_1) = -31.7883\,\text{deg}$$

$$I_2 := I_1 \qquad I_0 := I_1$$

$$I_{abc} := A_{012}\cdot\begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix}$$

$$\overrightarrow{|I_{abc}|} = \begin{pmatrix} 0.6927 \\ 0 \\ 0 \end{pmatrix}\text{pu} \qquad \overrightarrow{\arg(I_{abc})} = \begin{pmatrix} -31.7883 \\ 81.8699 \\ 81.8699 \end{pmatrix}\text{deg}$$

$V_{3new1} := E_2 + I_1 \cdot j \cdot \left(X_{1Mach} + X_T\right)$ $\qquad V_{3new1} = (0.9161 - 0.1354i)\,\text{pu}$

$V_{3new2} := 0 + I_2 \cdot j \cdot \left(X_{2Mach} + X_T\right)$ $\qquad V_{3new2} = (0.0341 + 0.055i)\,\text{pu}$

$V_{3new0} := 0 + I_0 \cdot j \cdot \left(X_{0Mach} + X_T + 3 \cdot X_{nMach}\right)$ $\qquad V_{3new0} = (0.0328 + 0.053i)\,\text{pu}$

$$V_{3newABC} := A_{012} \cdot \begin{pmatrix} V_{3new0} \\ V_{3new1} \\ V_{3new2} \end{pmatrix} \qquad \overrightarrow{\left|V_{3newABC}\right|} = \begin{pmatrix} 0.9833 \\ 0.9046 \\ 0.9008 \end{pmatrix} \text{pu} \qquad \overrightarrow{\arg\left(V_{3newABC}\right)} = \begin{pmatrix} -1.6028 \\ -132.1553 \\ 107.9302 \end{pmatrix} \text{deg}$$

$$\Delta V_{ABC} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} - V_{3newABC} \qquad \overrightarrow{\left|\Delta V_{ABC}\right|} = \begin{pmatrix} 0.0324 \\ 0.2229 \\ 0.2229 \end{pmatrix} \text{pu} \qquad \overrightarrow{\arg\left(\Delta V_{ABC}\right)} = \begin{pmatrix} 58.2117 \\ -61.2742 \\ 177.6976 \end{pmatrix} \text{deg}$$

Find the voltage across the two phase open fault:

$$V_{1bcopen} := I_{trans} \cdot \left[\frac{Z_{1pp'} \cdot \left(Z_{2pp'} + Z_{0pp'}\right)}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}}\right] \qquad \left|V_{1bcopen}\right| = 0.4041\,\text{pu} \qquad \arg\left(V_{1bcopen}\right) = 58.2117\,\text{deg}$$

$$V_{2bcopen} := I_{trans} \cdot \left(\frac{-Z_{1pp'} \cdot Z_{2pp'}}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}}\right) \qquad \left|V_{2bcopen}\right| = 0.1639\,\text{pu} \qquad \arg\left(V_{2bcopen}\right) = -121.7883\,\text{deg}$$

$$V_{0bcopen} := I_{trans} \cdot \left(\frac{-Z_{1pp'} \cdot Z_{0pp'}}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}}\right) \qquad \left|V_{0bcopen}\right| = 0.2401\,\text{pu} \qquad \arg\left(V_{0bcopen}\right) = -121.7883\,\text{deg}$$

$$\Delta V_{3\_1bcopen} := \left(\frac{Z_{bus1_{3,2}} - Z_{bus1_{3,3}}}{j \cdot X_{L1}}\right) \cdot V_{1bcopen} \qquad \left|\Delta V_{3\_1bcopen}\right| = 0.1593\,pu \qquad \arg\left(\Delta V_{3\_1bcopen}\right) = -121.7883\,deg$$

$$\Delta V_{3\_2bcopen} := \left(\frac{Z_{bus2_{3,2}} - Z_{bus2_{3,3}}}{j \cdot X_{L2}}\right) \cdot V_{2bcopen} \qquad \left|\Delta V_{3\_2bcopen}\right| = 0.0647\,pu \qquad \arg\left(\Delta V_{3\_2bcopen}\right) = 58.2117\,deg$$

$$\Delta V_{3\_0bcopen} := \left(\frac{Z_{bus0_{3,2}} - Z_{bus0_{3,3}}}{j \cdot X_{L0}}\right) \cdot V_{0bcopen} \qquad \left|\Delta V_{3\_0bcopen}\right| = 0.0623\,pu \qquad \arg\left(\Delta V_{3\_0bcopen}\right) = 58.2117\,deg$$

$$\Delta V_{3ABC\_bcopen} := A_{012} \cdot \begin{pmatrix} \Delta V_{3\_0bcopen} \\ \Delta V_{3\_1bcopen} \\ \Delta V_{3\_2bcopen} \end{pmatrix} \qquad \overrightarrow{\left|\Delta V_{3ABC\_bcopen}\right|} = \begin{pmatrix} 0.0324 \\ 0.2229 \\ 0.2229 \end{pmatrix} pu \qquad \overrightarrow{\arg\left(\Delta V_{3ABC\_bcopen}\right)} = \begin{pmatrix} -121.7883 \\ 118.7258 \\ -2.3024 \end{pmatrix} deg$$

Then using the Zbus method, the voltage at bus 3 would be:

$$V_{3ABC\_bcopen} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} + \Delta V_{3ABC\_bcopen} \qquad \overrightarrow{\left|V_{3ABC\_bcopen}\right|} = \begin{pmatrix} 0.9833 \\ 0.9046 \\ 0.9008 \end{pmatrix} pu \qquad \arg\left(V_{3ABC\_bcopen}\right) = \begin{pmatrix} -1.6028 \\ -132.1553 \\ 107.9302 \end{pmatrix} deg$$

Note the plus sign instead of the minus
sign as in the circuit based case above.

ATP Simulation Results: