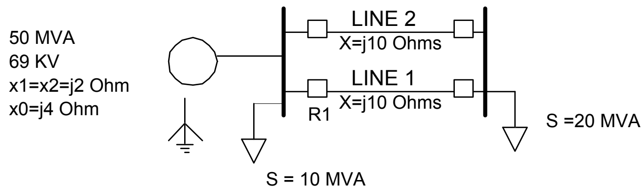
Do the following for the system below:



A. Sketch per unit circuits for the system in the sequence domain. Use $V_{b\text{-}ll}$ = 69kV and $S_B^{3\phi}$ = 50 MVA. Assume the lines have equal impedance in positive and negative sequence and three times the reactance in the zero sequence.

Define units: $MVA := 1000kW$ $MW := MVA$ $MVAr := MVA$

Define transformation $a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$ $A1 := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

$Vb := 69kV$ $Sb := 50MVA$ $Vbln := \frac{Vb}{\sqrt{3}}$

$Zb := \frac{Vb^2}{Sb}$ $Zb = 95.22\,\Omega$ $Ib := \frac{Sb}{3 \cdot Vbln}$

System impedances in ohms:

$Zsrc0 := j \cdot 4ohm$ $Zline1\_0 := j \cdot 30ohm$ $Zline2\_0 := j \cdot 30ohm$

$Zsrc1 := j \cdot 2ohm$ $Zline1\_1 := j \cdot 10ohm$ $Zline2\_1 := j \cdot 10ohm$

$Zsrc2 := Zsrc1$ $Zline1\_2 := j \cdot 10ohm$ $Zline2\_2 := j \cdot 10ohm$

System Impedances in per unit

$Zsrc0pu := \frac{Zsrc0}{Zb}$ $Zline1\_0pu := \frac{Zline1\_0}{Zb}$ $Zline2\_0pu := \frac{Zline2\_0}{Zb}$

$Zsrc0pu = 0.042i$ $Zline1\_0pu = 0.3151i$ $Zline2\_0pu = 0.3151i$

$Zsrc1pu := \frac{Zsrc1}{Zb}$ $Zline1\_1pu := \frac{Zline1\_1}{Zb}$ $Zline2\_1pu := \frac{Zline2\_1}{Zb}$

$Zsrc1pu = 0.021i$ $Zline1\_1pu = 0.105i$ $Zline2\_1pu = 0.105i$

$Zsrc2pu := Zsrc1pu$ $Zline1\_2pu := Zline1\_1pu$ $Zline2\_2pu := Zline2\_1pu$

Loads (model as series impedance)

$Sload1 := 10MVA$

$pfload1 := 0.8$

$$magZld1 := \frac{Vb^2}{Sload1}$$

$\theta 1 := acos(0.8)$ $\theta 1 = 36.8699\,deg$

$Zload1 := magZld1 \cdot e^{j \cdot \theta 1}$

$Rload1 := Re(Zload1)$ $Rload1 = 380.88\,\Omega$

$Xload1 := Im(Zload1)$ $Xload1 = 285.66\,\Omega$

$Sload2 := 20MVA$

$pfload2 := 0.8$

$$magZld2 := \frac{Vb^2}{Sload2}$$

$\theta 2 :=$ a $\theta 2 = 36.8699\,deg$

$Zload2 := magZld2 \cdot e^{j \cdot \theta 2}$

$Rload2 := Re(Zload2)$ $Rload2 = 190.44\,\Omega$

$Xload2 := Im(Zload2)$ $Xload2 = 142.83\,\Omega$

Model as being the same in positive and negative sequence, and open (ungrounded) in zero sequence)

$Rld1\_pu\_pos := \frac{Rload1}{Zb}$ $Rld1\_pu\_pos = 4$

$Rld1\_pu\_neg := Rld1\_pu\_pos$

$Xld1\_pu\_pos := \frac{Xload1}{Zb}$ $Xld1\_pu\_pos = 3$

$Xld1\_pu\_neg := Xld1\_pu\_pos$

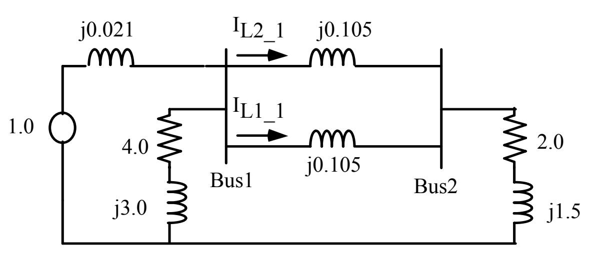
$Rld2\_pu\_pos := \frac{Rload2}{Zb}$ $Rld2\_pu\_pos = 2$

$Rld2\_pu\_neg := Rld2\_pu\_pos$

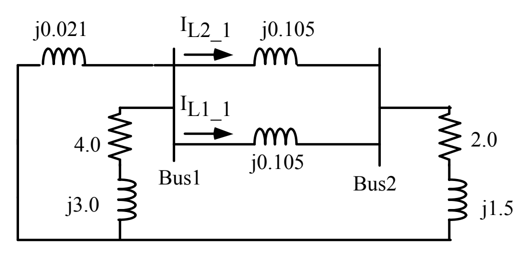
$Xld2\_pu\_pos := \frac{Xload2}{Zb}$ $Xld2\_pu\_pos = 1.5$

$Xld2\_pu\_neg := Xld2\_pu\_pos$

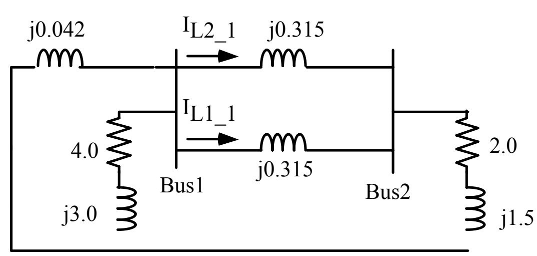
Positive sequence equivalent:



Negative sequence equivalent:



Zero Sequence Equivalent:
note that the loads are open.



B. Compute the voltages and currents seen at point R1 for the following faults located at 10% and 90% of the way down line 1 from the source end: SLG, 3ϕ, LL, DLG, and Phase "a" open. Also sketch phasor diagrams. Set the fault impedances at zero.

First calculate sequence equivalent impedances to the fault (with loads neglected)

Positive Sequence:

$$ZL1\_left\_pos := 0.1 \cdot Zline1\_1pu \qquad ZL1\_left\_pos = 0.0105i$$

$$ZL1\_right\_pos := 0.9 \cdot Zline1\_1pu \qquad ZL1\_right\_pos = 0.0945i$$

$$Zthev1 := Zsrc1pu + \left[\frac{1}{ZL1\_left\_pos} + \left(\frac{1}{Zline2\_1pu + ZL1\_right\_pos}\right)\right]^{-1} \qquad Zthev1 = 0.031i$$

Negative Sequence:

$$ZL1\_left\_neg := 0.1 \cdot Zline1\_2pu \qquad ZL1\_left\_neg = 0.0105i$$

$$ZL1\_right\_neg := 0.9 \cdot Zline1\_2pu \qquad ZL1\_right\_neg = 0.0945i$$

$$Zthev2 := Zsrc2pu + \left[\frac{1}{ZL1\_left\_neg} + \left(\frac{1}{Zline2\_2pu + ZL1\_right\_neg}\right)\right]^{-1} \qquad Zthev2 = 0.031i$$

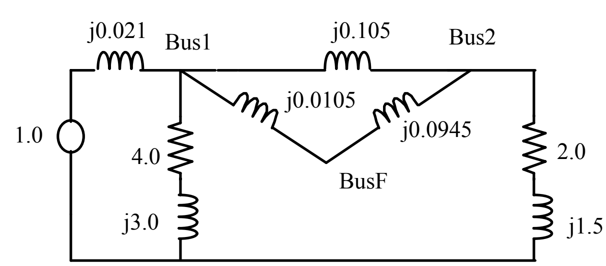
Zero Sequence:

$$ZL1\_left\_0 := 0.1 \cdot Zline1\_0pu \qquad ZL1\_left\_0 = 0.0315i$$

$$ZL1\_right\_0 := 0.9 \cdot Zline1\_0pu \qquad ZL1\_right\_0 = 0.2836i$$

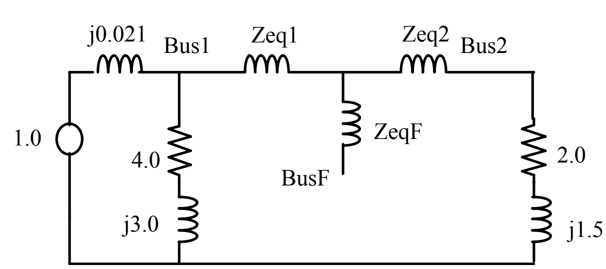
$$Zthev0 := Zsrc0pu + \left[\frac{1}{ZL1\_left\_0} + \left(\frac{1}{Zline2\_0pu + ZL1\_right\_0}\right)\right]^{-1} \qquad Zthev0 = 0.0719i$$

Now repeat the calculations with the loads included (the zero sequence won't change) to see how much impact the loads will have on the calculations.



Positive sequence:

The fault bascially creates a Delta in the middle of the system (the corners of the delta are the buses at the end of the lines and the fault point). Converte this to a Y to make it easier to add in the loads



$$Zeq1\_1 := \frac{Zline2\_1pu \cdot 0.1 \cdot Zline1\_1pu}{Zline2\_1pu + 0.1 \cdot Zline1\_1pu + 0.9 \cdot Zline1\_1pu} \qquad Zeq1\_1 = 5.251i \times 10^{-3}$$

$$Zeq2\_1 := \frac{Zline2\_1pu \cdot 0.9 \cdot Zline1\_1pu}{Zline2\_1pu + 0.1 \cdot Zline1\_1pu + 0.9 \cdot Zline1\_1pu} \qquad Zeq2\_1 = 0.0473i$$

$$Zeqf\_1 := \frac{0.1Zline1\_1pu \cdot 0.9 \cdot Zline1\_1pu}{Zline2\_1pu + 0.1 \cdot Zline1\_1pu + 0.9 \cdot Zline1\_1pu} \qquad Zeqf\_1 = 4.7259i \times 10^{-3}$$

If there is no fault, then the equivalent for the line is

$Zeq1\_1 + Zeq2\_1 = 0.0525i$ hich is $\frac{Zline1\_1pu}{2} = 0.0525i$ as expected.

So the positive sequence impedance looking in from BusF is made up Zeqf_1 plus the parallel impedances of the other paths:

$$Zleft\_1 := Zeq1\_1 + \left(\frac{1}{Zsrc1pu} + \frac{1}{Rld1\_pu\_pos + j \cdot Xld1\_pu\_pos}\right)^{-1} \qquad Zleft\_1 = 7.0232 \times 10^{-5} + 0.0262i$$

$$Zright\_1 := Zeq2\_1 + Rld2\_pu\_pos + j \cdot Xld2\_pu\_pos$$

$$Zequiv1\_act := Zeqf\_1 + \left(\frac{1}{Zleft\_1} + \frac{1}{Zright\_1}\right)^{-1} \qquad Zequiv1\_act = 2.8137 \times 10^{-4} + 0.0308i$$

and earlier we found $Zthev1 = 0.031i$

So there is a small difference:

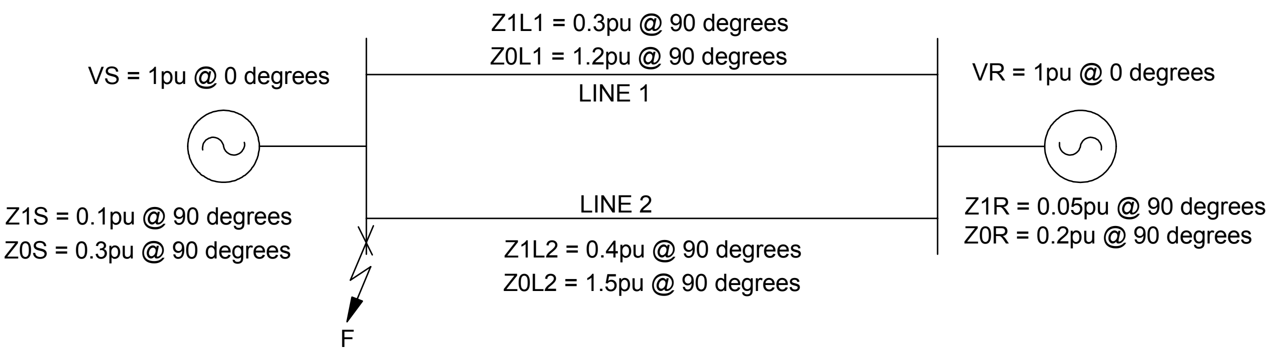
$$Error := \frac{|Zequiv1\_act| - |Zthev1|}{|Zequiv1\_act|} \qquad Error = -0.7142\,\%$$

This is a smaller error than there is in the model data.

The negative sequence impedances will be the same.

So for all calculations expect the phase A open, the loads will be neglected

4. For the following system with all quantities shown in per unit:
   a. Develop the each of the sequence networks for a fault at location F.
   b. Develop the sequence network connections for single-phase to ground, phase-phase to ground, phase-phase, and three-phase faults.
   c. Determine the positive-, negative-, and zero-sequence currents and voltages at location F for a single-phase to ground fault.



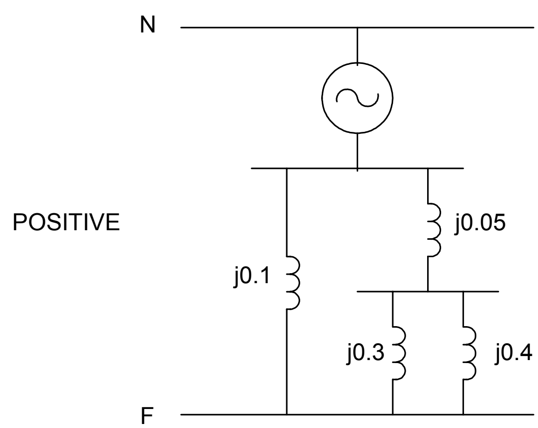
$a := 1 \cdot e^{j \cdot 120deg}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$pu := 1$

$ORIGIN := 1$

Solution for 4a: Develop the each of the sequence networks for a fault at location F.



$Z_{1S} := j \cdot 0.1pu$ $\qquad Z_{1R} := j \cdot 0.05pu$

$Z_{1L1} := j \cdot 0.3$ $\qquad Z_{1L2} := j \cdot 0.4$

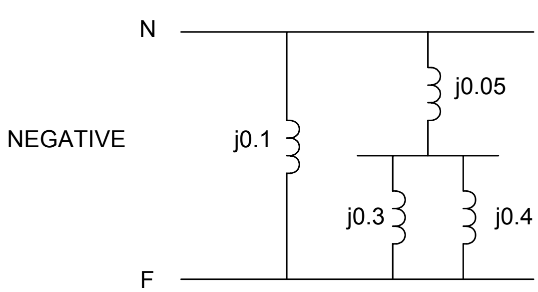
$$Z_{1LineEq} := \left(\frac{1}{Z_{1L1}} + \frac{1}{Z_{1L2}} \right)^{-1}$$

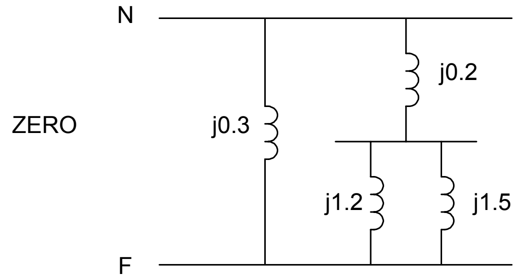
$Z_{1LineEq} = 0.1714i\,pu$

$$Z_1 := \left(\frac{1}{Z_{1S}} + \frac{1}{Z_{1LineEq} + Z_{1R}} \right)^{-1}$$

$Z_1 = 0.0689i\,pu$

$Z_2 := Z_1$





$Z_{0S} := j \cdot 0.3pu \qquad Z_{0R} := j \cdot 0.2pu$

$Z_{0L1} := j \cdot 1.2 \qquad Z_{0L2} := j \cdot 1.5$

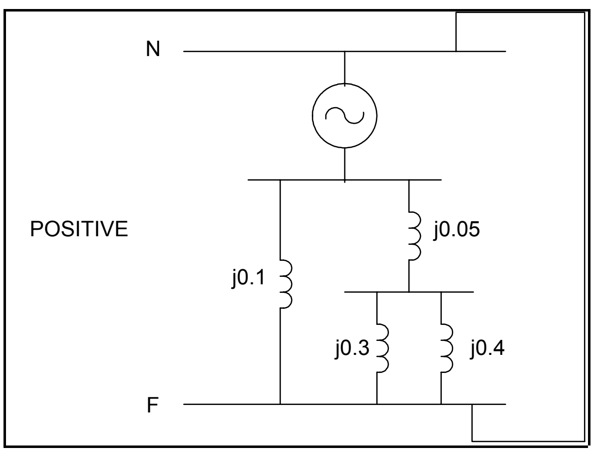
$$Z_{0LineEq} := \left(\frac{1}{Z_{0L1}} + \frac{1}{Z_{0L2}} \right)^{-1}$$

$$Z_{0LineEq} = 0.6667i\,pu$$

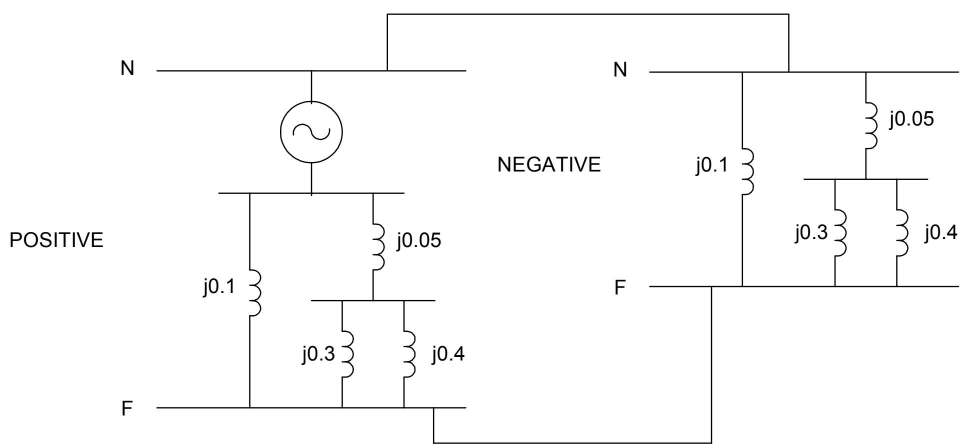
$$Z_0 := \left(\frac{1}{Z_{0S}} + \frac{1}{Z_{0LineEq} + Z_{0R}} \right)^{-1}$$

$$Z_0 = 0.2229i\,pu$$

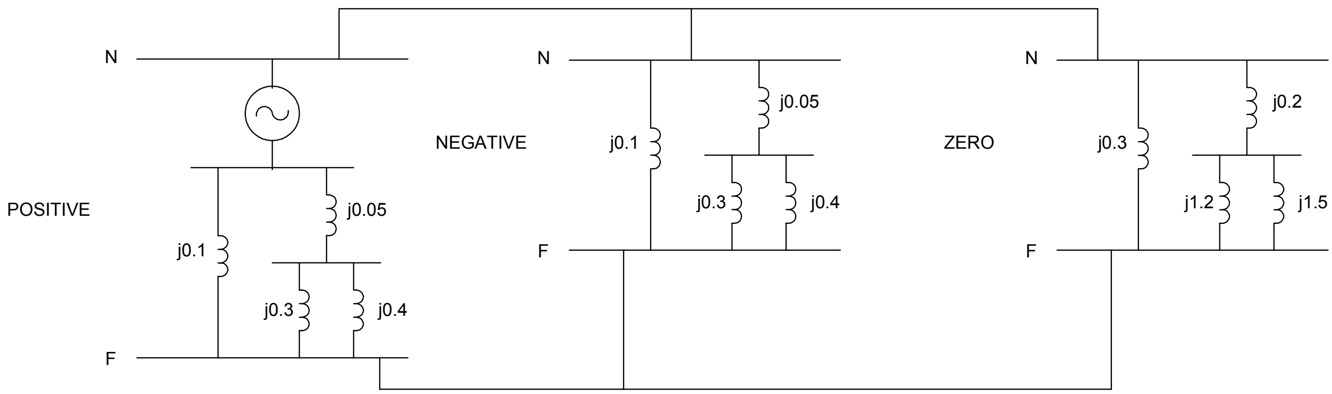
Solution for 4b): Three Phase Fault



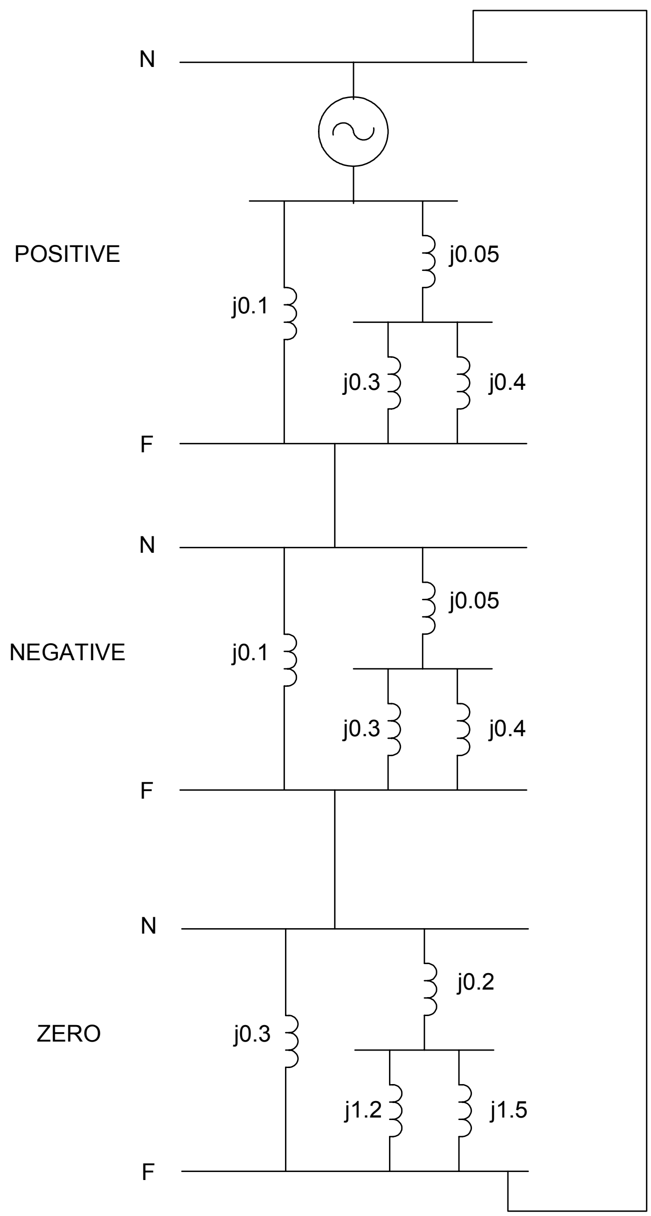
Problem 4b: Line to line fault:



Problem 4b: Double Line to Ground Fault:



Problem 4b: Single line to ground fault:



c. Determine the positive-, negative-, and zero-sequence currents and voltages at location F for a single-phase to ground fault.

Assuming the fault is on phase A:

$I_{0F} := \dfrac{1.0}{Z_1 + Z_2 + Z_0}$ $\quad I_{0F} = -2.7729i\,pu$ $\quad I_{1F} := I_{0F}$ $\quad I_{2F} := I_{0F}$

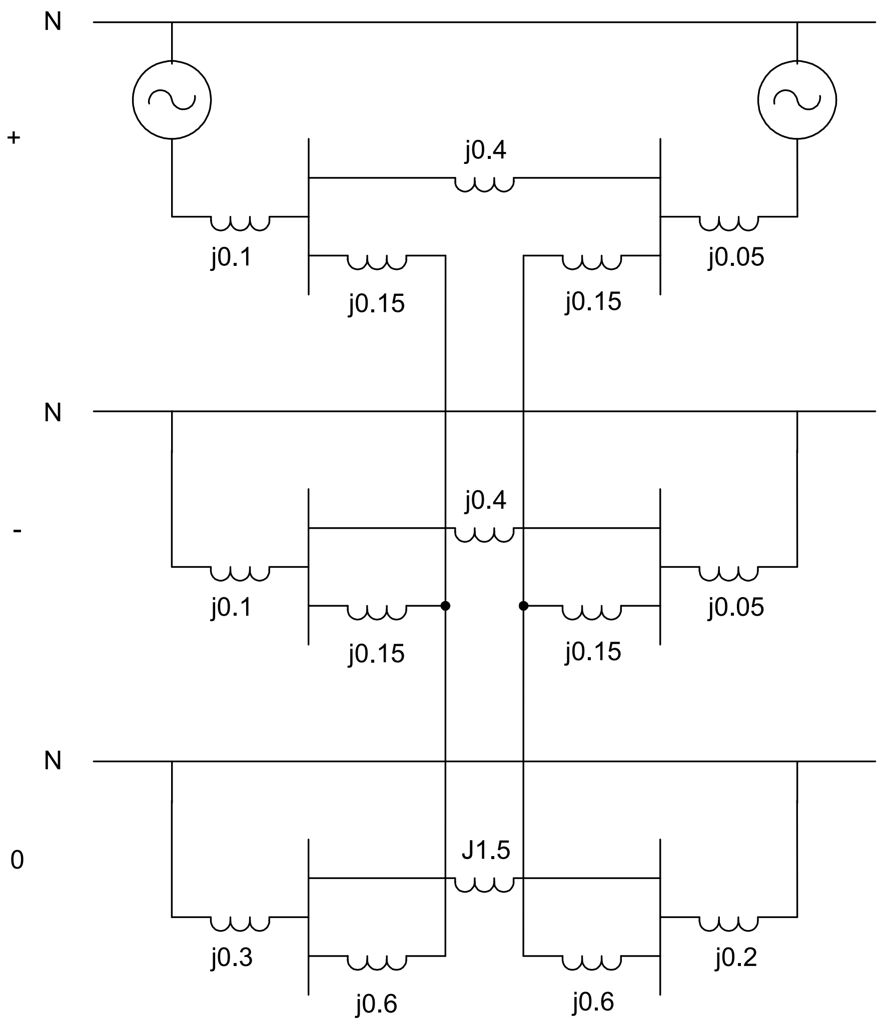
$I_{AF} := I_{0F} + I_{1F} + I_{0F}$ $\quad I_{AF} = -8.3187i\,pu$

$V_0 := -I_{0F} \cdot Z_0$ $\quad \left|V_0\right| = 0.618\,pu$ $\quad \arg(V_0) = 180\,deg$

$V_1 := 1.0 - I_{1F} \cdot Z_1$ $\quad \left|V_1\right| = 0.809\,pu$ $\quad \arg(V_1) = 0\,deg$

$V_2 := -I_{2F} \cdot Z_2$ $\quad \left|V_2\right| = 0.191\,pu$ $\quad \arg(V_2) = 180\,deg$

5. For the system shown in 4, develop the sequence connection diagram for a single-phase open on Line 1.



$$Z_{1equiv} := \frac{Z_{1L1}}{2} + \left(\frac{1}{Z_{1S} + Z_{1R}} + \frac{1}{Z_{1L2}}\right)^{-1} + \frac{Z_{1L1}}{2} \qquad Z_{1equiv} = 0.4091i\,pu$$

$$Z_{2equiv} := Z_{1equiv}$$

$$Z_{0equiv} := Z_{0L1} + \left(\frac{1}{Z_{0S} + Z_{0R}} + \frac{1}{Z_{0L2}}\right)^{-1} \qquad Z_{0equiv} = 1.575i\,pu$$

6. Using the sequence diagram from problem 5, calculate the positive-, negative-, and zero-sequence currents on Line 2 with VR = 1pu @ 20 degrees.

$$V_S := 1pu \cdot e^{j \cdot 0deg}$$

$$V_R := 1pu \cdot e^{j \cdot 20deg}$$

Total prefault current:

$$I_{sourceprefault} := \frac{V_S - V_R}{Z_{1S} + Z_{1R} + \left(\frac{1}{Z_{1L1}} + \frac{1}{Z_{1L2}}\right)^{-1}} \qquad \left|I_{sourceprefault}\right| = 1.0805\,pu \qquad \arg\left(I_{sourceprefault}\right) = -170\,deg$$

Current dividers to find the current in lines 1 and 2:

$$I_{L1\_prefault} := I_{sourceprefault} \cdot \left(\frac{Z_{1L2}}{Z_{1L1} + Z_{1L2}}\right) \qquad \left|I_{L1\_prefault}\right| = 0.6174\,pu \qquad \arg\left(I_{L1\_prefault}\right) = -170\,deg$$

$$I_{L2\_prefault} := I_{sourceprefault} \cdot \left(\frac{Z_{1L1}}{Z_{1L1} + Z_{1L2}}\right) \qquad \left|I_{L2\_prefault}\right| = 0.4631\,pu \qquad \arg\left(I_{L2\_prefault}\right) = -170\,deg$$

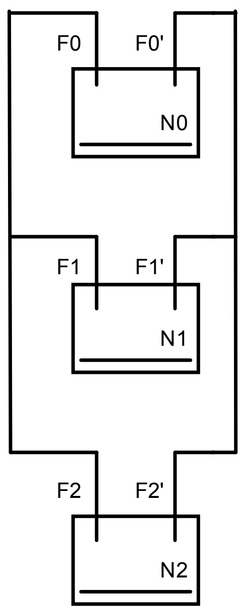
$$I_{ABCL1\_prefault} := I_{L1\_prefault} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{ABCL1\_prefault}\right|} = \begin{pmatrix} 0.6174 \\ 0.6174 \\ 0.6174 \end{pmatrix} pu$$

$$\overrightarrow{\arg\left(I_{ABCL1\_prefault}\right)} = \begin{pmatrix} -170 \\ 70 \\ -50 \end{pmatrix} deg$$

$$I_{ABCL2\_prefault} := I_{L2\_prefault} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{ABCL2\_prefault}\right|} = \begin{pmatrix} 0.4631 \\ 0.4631 \\ 0.4631 \end{pmatrix} pu$$

$$\overrightarrow{\arg(I_{ABCL2\_prefault})} = \begin{pmatrix} -170 \\ 70 \\ -50 \end{pmatrix} \deg$$

Phase A open analysis:



Equivalent voltage source for phase A open analysis:

$V_{se} := V_S - V_R$ $\qquad V_{se} = 0.0603 - 0.342i\,pu$

Norton Equivalent Current: $\quad I_{se} := \dfrac{V_{se}}{Z_{1R} + Z_{1S}}$

$I_{se} = -2.2801 - 0.402i\,pu$

Equivalent Parallel Impedance: $\quad Z_{eq} := \left(\dfrac{1}{Z_{1L2}} + \dfrac{1}{Z_{1S} + Z_{1R}}\right)^{-1}$

$Z_{eq} = 0.1091i\,pu$

Convert back to Thevenin Equivalent Voltage

$V_f := Z_{eq} \cdot I_{se}$ $\qquad |V_f| = 0.2526\,pu \quad \arg(V_f) = -80\,\deg$

Positive sequence current in line 1:

$$I_{1L1\_open} := \frac{V_f}{Z_{1equiv} + \left(\dfrac{1}{Z_{2equiv}} + \dfrac{1}{Z_{0equiv}}\right)^{-1}}$$

$|I_{1L1\_open}| = 0.3442\,pu$

$\arg(I_{1L1\_open}) = -170\,\deg$

Negative sequence current in line 1 (current divider on the line 1 current)

$$I_{2L1\_open} := -I_{1L1\_open} \cdot \frac{Z_{0equiv}}{Z_{2equiv} + Z_{0equiv}}$$

$|I_{2L1\_open}| = 0.2732\,pu$

$\arg(I_{2L1\_open}) = 10\,\deg$

Zero sequence current in line 1 (current divider on the line 1 current)

$$I_{0L1\_open} := -I_{1L1\_open} \cdot \frac{Z_{2equiv}}{Z_{2equiv} + Z_{0equiv}}$$

$$\left|I_{0L1\_open}\right| = 0.071\,pu$$

$$\arg\left(I_{0L1\_open}\right) = 10\,deg$$

$$I_{ABC\_Line1} := A_{012} \cdot \begin{pmatrix} I_{0L1\_open} \\ I_{1L1\_open} \\ I_{2L1\_open} \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_Line1}\right|} = \begin{pmatrix} 0 \\ 0.5452 \\ 0.5452 \end{pmatrix} pu$$

$$\arg\left(I_{ABC\_Line1_2}\right) = 88.7404\,deg$$

$$\arg\left(I_{ABC\_Line1_3}\right) = -68.7404\,deg$$

Note that the magnitude on phase A is 0 and a little smaller on the unfaulted phases. There is also a phase shift compared to prefault

Now to fine the line 2 current, we need to do another current divider on each of the sequence currents from line 1, since the sequnce currents could either pass through the sources or line to to return to line 1.

Positive sequence load current in line two ignoring open line

$$I_{LineA2} := \frac{V_S - V_R}{Z_{1S} + Z_{1L2} + Z_{1R}}$$

$$\left|I_{LineA2}\right| = 0.631$$

$$\arg\left(I_{LineA2}\right) = -170\,deg$$

$$I_{1L2} := -I_{1L1\_open} \cdot \left(\frac{Z_{1S} + Z_{1R}}{Z_{1S} + Z_{1R} + Z_{1L2}}\right) + I_{LineA2}$$

$$\left|I_{1L2}\right| = 0.5376\,pu \qquad \arg\left(I_{1L2}\right) = -170\,deg$$

$$I_{2L2} := -I_{2L1\_open} \cdot \left(\frac{Z_{1S} + Z_{1R}}{Z_{1S} + Z_{1R} + Z_{1L2}}\right)$$

$$\left|I_{2L2}\right| = 0.0745\,pu \qquad \arg\left(I_{2L2}\right) = -170\,deg$$

$$I_{0L2} := -I_{0L1\_open} \cdot \left(\frac{Z_{0S} + Z_{0R}}{Z_{0S} + Z_{0R} + Z_{0L2}}\right)$$

$$\left|I_{0L2}\right| = 0.0177\,pu \qquad \arg\left(I_{0L2}\right) = -170\,deg$$

$$I_{ABC\_Line2} := A_{012} \cdot \begin{pmatrix} I_{0L2} \\ I_{1L2} \\ I_{2L2} \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_Line2}\right|} = \begin{pmatrix} 0.6298 \\ 0.4939 \\ 0.4939 \end{pmatrix} pu \qquad \overrightarrow{\arg\left(I_{ABC\_Line2}\right)} = \begin{pmatrix} -170 \\ 64.2868 \\ -44.2868 \end{pmatrix} deg$$