

Phasors, Single and Three Phase Power

1. Notation for instantaneous and phasor quantities:

(a) Instantaneous: $v(t) = V_{max} \cos(\omega t + \theta_v)$

(b) RMS Phasor quantity: $\bar{V} = \frac{V_{max}}{\sqrt{2}} \angle \theta_v$

2. Instantaneous single phase power: $p(t) = \frac{1}{2} V_{max} I_{max} [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$

3. Average single phase power: $P_{av} = \frac{1}{2} V_{max} I_{max} [\cos(\theta_v - \theta_i)]$

4. Power Factor

(a) Power factor angle (ϕ) = $\theta_v - \theta_i$

(b) $-90^\circ \leq \phi \leq 90^\circ$

(c) Power factor: $pf = \cos(\phi)$, $0 \leq pf \leq 1$

(d) Lagging Power Factor: $\phi > 0$

(e) Leading Power Factor: $\phi < 0$

5. Three Phase Power

(a) $\bar{S}_{3\phi} = 3\bar{V}_\phi \bar{I}_\phi^*$
 $|\bar{S}_{3\phi}| = \sqrt{3} |\bar{V}_{ll}| |\bar{I}_l^*|$

(b) $P = 3|\bar{V}_\phi| |\bar{I}_\phi| \cos\phi$

(c) $Q = 3|\bar{V}_\phi| |\bar{I}_\phi| \sin\phi$

(d) $P = \sqrt{3} |\bar{V}_{ll}| |\bar{I}_l| \cos\phi$ (ϕ is the angle between **Phase** quantities!)

(e) $Q = \sqrt{3} |\bar{V}_{ll}| |\bar{I}_l| \sin\phi$

(f) Power dissipated in shunt resistor (Y connected):

$$P = \frac{3|\bar{V}_\phi|^2}{R} = \frac{|\bar{V}_{ll}|^2}{R}$$

(g) Reactive power to/from Y connected shunt reactance (capacitor or reactor)

$$Q = \frac{3|\bar{V}_\phi|^2}{X} = \frac{|\bar{V}_{ll}|^2}{X}$$

6. $\bar{Z}_\Delta = 3\bar{Z}_Y$

7. The “a” operator:

(a) $\bar{a} = 1 \angle 120^\circ$

(b) $\bar{a}^2 = 1 \angle 240^\circ$

- (c) $\overline{a^3} = 1 \angle 360^\circ$
- (d) $1 + \overline{a} + \overline{a^2} = 0$
- (e) $1 - \overline{a} = \sqrt{3} \angle -30^\circ$
- (f) $1 - \overline{a^2} = \sqrt{3} \angle 30^\circ$
- (g) $\overline{a} - 1 = \sqrt{3} \angle +150^\circ$
- (h) $\overline{a^2} - 1 = \sqrt{3} \angle -150^\circ$
- (i) $\overline{a^2} - \overline{a} = \sqrt{3} \angle -90^\circ$
- (j) $\overline{a} - \overline{a^2} = \sqrt{3} \angle 90^\circ$
- (k) $\overline{a^2} + \overline{a} = 1 \angle 180^\circ$
- (l) $\overline{I_b} = \overline{a^2 I_a}$
- (m) $\overline{I_c} = \overline{a I_a}$