Input Signals

$PTCT :=$

C:\..\EE504P1xam.txt

$PT := \frac{120}{138000}$ $CT := \frac{5}{3000}$

$NN := rows(PTCT)$ Size of arrays

$n := 0 .. NN$ Sample array (for plotting)

$time := submatrix(PTCT, 0, NN - 1, 0, 0)$

$Va := submatrix(PTCT, 0, NN - 1, 1, 1) \cdot PT$

$Ia := submatrix(PTCT, 0, NN - 1, 2, 2) \cdot CT$

$Vb := submatrix(PTCT, 0, NN - 1, 3, 3) \cdot PT$

$Ib := submatrix(PTCT, 0, NN - 1, 4, 4) \cdot CT$

$Vc := submatrix(PTCT, 0, NN - 1, 5, 5) \cdot PT$

$Ic := submatrix(PTCT, 0, NN - 1, 6, 6) \cdot CT$

Digital Filter Constants

Cosine Filter

$Ndft := 16$ $nf := 0 .. Ndft - 1$

$Zd_{nf} := 0$ $Zd_0 := \frac{Ndft}{2}$

$Zn_{nf} := Re\left(e^{-i \cdot 2 \cdot \pi \cdot \frac{nf}{Ndft}}\right)$

Delayed Cosine Filter

$Znim := 0 .. \left(\frac{Ndft}{4}\right)$ $Zdim := 0 .. \left(\frac{Ndft}{4}\right)$

$Znim_{\left(\frac{Ndft}{4}\right)} := 1$ $Zdim_0 := 1$

Memory Filter

$K := 15$ Filter time constant

$Zmemn := (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$

$Zmemd := [K\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ (K - 1)]^T$

Digital Filtering Subroutine

"x" is an array of real numbers containing the

$digitalfilter(x, Zn, Zd) :=$
$Nd \leftarrow rows(Zd)$
$Nn \leftarrow rows(Zn)$

numbers containing the
input data.

Zn is a vector of coefficients for the filter numerator polynomial in dessending powers of 1/z.

Zd is a vector of coefficients for the filter denominator polynomial in desending powers of 1/z.

Returned vector is the filtered data.

$$
\begin{array}{l}
Nx \leftarrow rows(x) \\
\text{for } k \in 0..Nx \\
\quad y_k \leftarrow 0 \\
jn \leftarrow 0 \\
jd \leftarrow 0 \\
\text{for } n \in 0..Nx-1 \\
\quad \left| \begin{array}{l}
z_0 \leftarrow x_n \\
y_n \leftarrow 0 \\
\text{for } k \in 0..jn \\
\quad y_n \leftarrow y_n + x_{n-k} \cdot Zn_k \\
\text{for } k \in 1..jd \\
\quad y_n \leftarrow y_n - Zd_k \cdot y_{n-k} \;\; \text{if } jd > 0 \\
jn \leftarrow jn + 1 \;\; \text{if } jn < Nn - 1 \\
jd \leftarrow jd + 1 \;\; \text{if } jd < Nd - 1 \\
y_n \leftarrow \dfrac{y_n}{Zd_0}
\end{array} \right. \\
n \leftarrow 0 \\
\text{return } y
\end{array}
$$

Compute peak amplitude Phase and Sequence voltages and currents

$N := \mathrm{rows}(Va) \quad n := 0 .. N - 1$

$Var := \mathrm{digitalfilter}(Va, Zn, Zd) \quad Vbr := \mathrm{digitalfilter}(Vb, Zn, Zd) \quad Vcr := \mathrm{digitalfilter}(Vc, Zn, Zd)$

$Vai := \mathrm{digitalfilter}(Var, Znim, Zdim) \quad Vbi := \mathrm{digitalfilter}(Vbr, Znim, Zdim) \quad Vci := \mathrm{digitalfilter}(Vcr, Znim, Zdim$

$Vav_n := Var_n + (i \cdot Vai)_n \quad Vbv_n := Vbr_n + (i \cdot Vbi)_n \quad Vcv_n := Vcr_n + (i \cdot Vci)_n$

$Vabc := \mathrm{augment}(Vav, Vbv) \quad Vabc := \mathrm{augment}(Vabc, Vcv) \quad V012 := \mathrm{symm3}(Vabc)$

$V0 := \mathrm{submatrix}(V012, 0, N - 1, 0, 0) \quad V0m_n := \left|V0_n\right|$

$V1 := \mathrm{submatrix}(V012, 0, N - 1, 1, 1) \quad V1m_n := \left|V1_n\right|$

$V2 := \mathrm{submatrix}(V012, 0, N - 1, 2, 2) \quad V2m_n := \left|V2_n\right|$

$Iar := \mathrm{digitalfilter}(Ia, \mathrm{Re}(Zn), Zd) \quad Ibr := \mathrm{digitalfilter}(Ib, \mathrm{Re}(Zn), Zd) \quad Icr := \mathrm{digitalfilter}(Ic, \mathrm{Re}(Zn), Zd)$

$Iai := \mathrm{digitalfilter}(Iar, Znim, Zdim) \quad Ibi := \mathrm{digitalfilter}(Ibr, Znim, Zdim) \quad Ici := \mathrm{digitalfilter}(Icr, Znim, Zdim)$

$Iav_n := Iar_n + (i \cdot Iai)_n \quad Ibv_n := Ibr_n + (i \cdot Ibi)_n \quad Icv_n := Icr_n + (i \cdot Ici)_n$

$Iabc := \mathrm{augment}(Iav, Ibv) \quad Iabc := \mathrm{augment}(Iabc, Icv) \quad I012 := \mathrm{symm3}(Iabc)$

$I0 := \mathrm{submatrix}(I012, 0, N - 1, 0, 0) \quad I0m_n := \left|I0_n\right|$

$I1 := \mathrm{submatrix}(I012, 0, N - 1, 1, 1) \quad I1m_n := \left|I1_n\right|$

$I2 := \mathrm{submatrix}(I012, 0, N - 1, 2, 2) \quad I2m_n := \left|I2_n\right|$

1.8 Distance element polarized by prefault positive sequence voltage

1.8.1 Sending end polorizing

$$Va1_n := \frac{\left(Vamem_n + a \cdot Vbmem_n + a^2 \cdot Vcmem_n\right)}{3}$$

$$Vb1_n := \frac{\left(Vbmem_n + a \cdot Vcmem_n + a^2 \cdot Vamem_n\right)}{3}$$

$$Vc1_n := \frac{\left(Vcmem_n + a \cdot Vamem_n + a^2 \cdot Vbmem_n\right)}{3}$$

$$k0 := \frac{(ZL0 - ZL1)}{3 \cdot ZL1}$$

$$ZL0 = 9.108 + 101.834i$$

$$ZL1 = 2.142 + 25.452i$$

$$k0 = 1.001 - 6.994i \times 10^{-3}$$

Ground Element Reach Calculations

$$\begin{aligned}
gnddist(V, I, Ires, Vp) := \; & N \leftarrow rows(V) \\
& \text{for } n \in (0 .. N-1) \\
& \quad Num \leftarrow \frac{Re\left(V_n \cdot \overline{Vp_n}\right)}{Re\left[ZL1 \cdot (I_n + k0 \cdot Ires_n) \cdot \overline{Vp_n}\right]} \text{ if } Re\left[(I_n + k0 \cdot Ires_n) \cdot \overline{Vp_n}\right] \neq 0 \\
& \quad Num \leftarrow 2 \text{ otherwise} \\
& \quad m_n \leftarrow Num \text{ if } |Num| < 2 \\
& \quad m_n \leftarrow \frac{2Num}{|Num|} \text{ otherwise} \\
& \text{return } m
\end{aligned}$$

Phase Element Reach Calculations

$$\text{phasedist}(V1, V2, I1, I2, Vp) := \left| \begin{array}{l} N \leftarrow \text{rows}(Vp) \\ \text{for } n \in (0 .. N-1) \\ \quad \left| \begin{array}{l} \text{Denom} \leftarrow \text{Re}\left[ZL1 \cdot (I1_n - I2_n) \cdot \left(\overline{-i\,Vp_n}\right)\right] \\ \text{Num} \leftarrow \dfrac{\text{Re}\left[(V1_n - V2_n) \cdot \left(\overline{-i \cdot Vp_n}\right)\right]}{\text{Denom}} \text{ if Denom} \neq 0 \\ \text{NUM} \leftarrow 2 \text{ otherwise} \\ \text{NUM} \leftarrow 2 \text{ if } \left|\text{Re}\left[(V1_n - V2_n) \cdot \left(\overline{-i \cdot Vp_n}\right)\right]\right| < 1 \\ m_n \leftarrow \text{Num if } |\text{Num}| \leq 2 \\ m_n \leftarrow 2 \text{ otherwise} \end{array} \right. \\ \text{return } m \end{array} \right.$$

1.8.3 Fault Distance

$Ires_n := Iav_n + Ibv_n + Icv_n$

$trip_n := 0.8$

$mag := \text{gnddist}(Vav, Iav, Ires, Va1)$

$mbg := \text{gnddist}(Vbv, Ibv, Ires, Vb1)$

$mcg := \text{gnddist}(Vcv, Icv, Ires, Vc1)$

Ground Elements

$mab := \text{phasedist}(Vav, Vbv, Iav, Ibv, Vc1)$

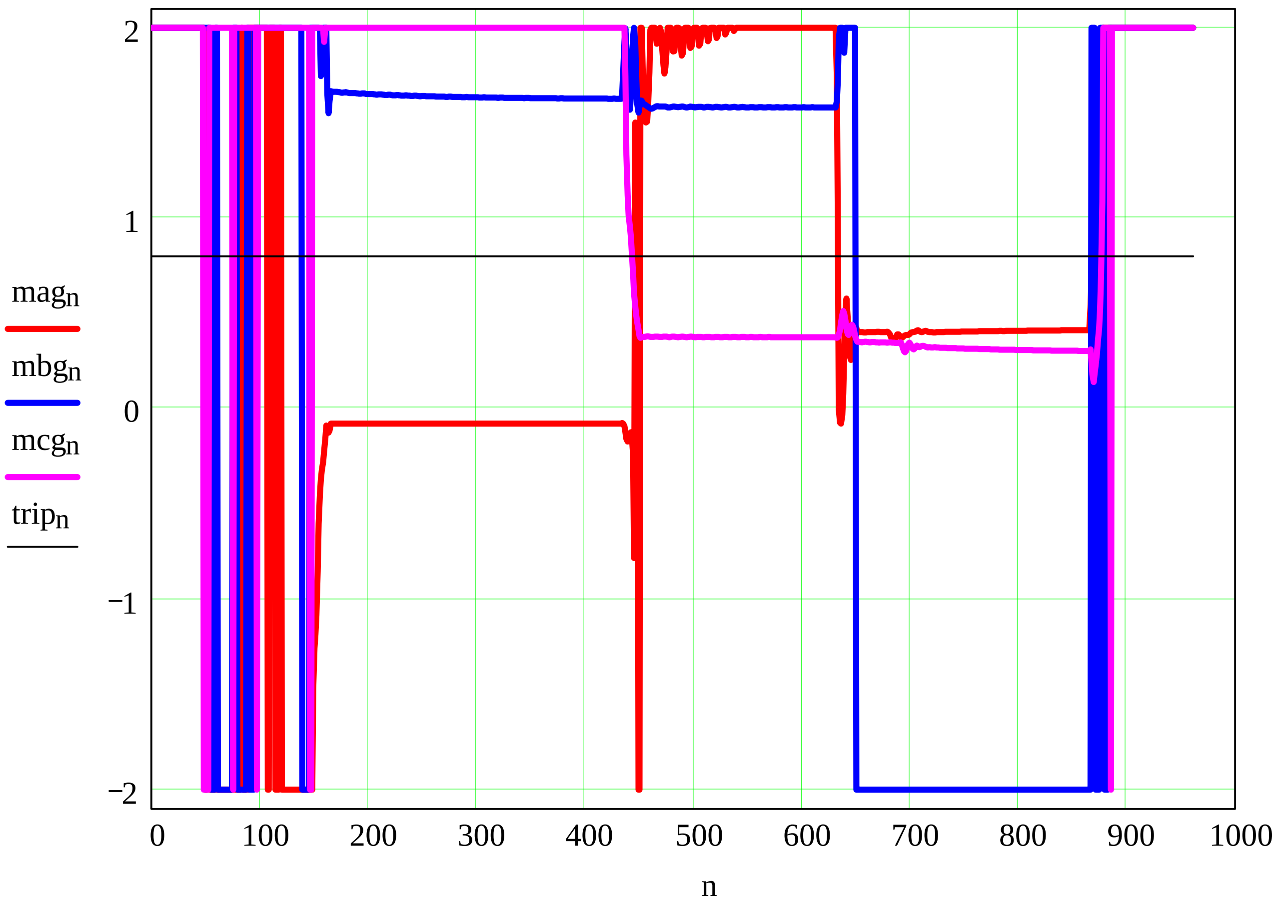
$mbc := \text{phasedist}(Vbv, Vcv, Ibv, Icv, Va1)$

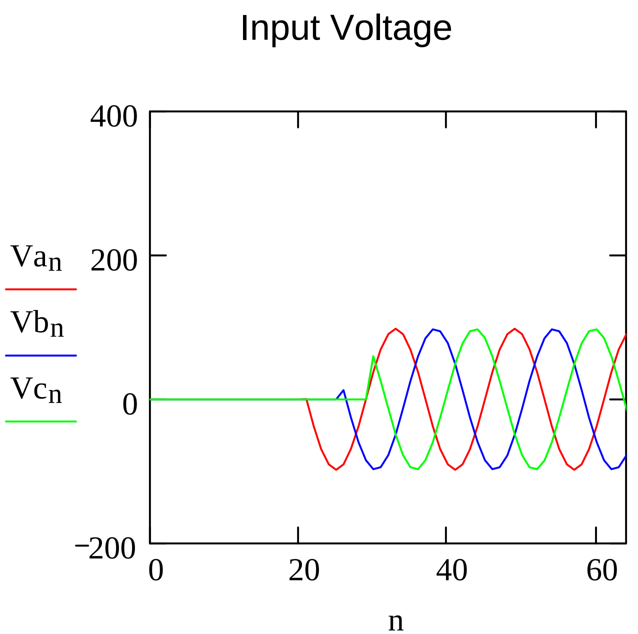
$mca := \text{phasedist}(Vcv, Vav, Icv, Iav, Vb1)$

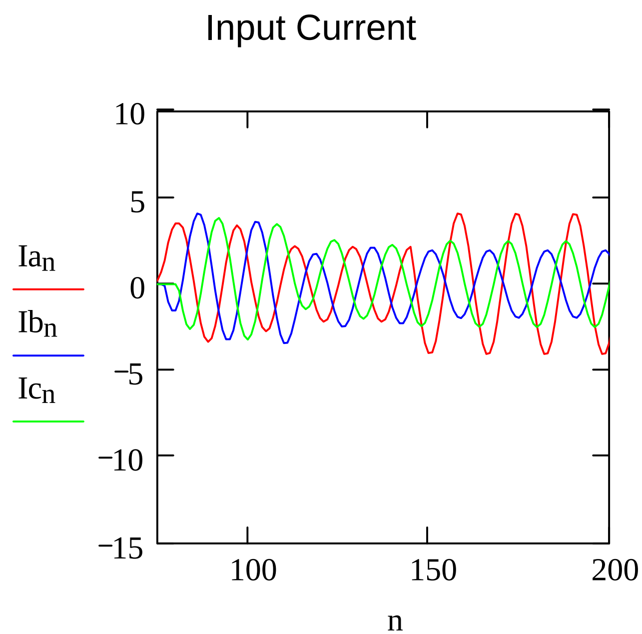
Phase Elements

$mca_{700} = 0.217$ $mcg_{700} = 0.339$ $mag_{700} = 0.397$

Ground Distance Reach







Symmetrical Component Cnstants

$$a := e^{i\cdot\pi\cdot\frac{2}{3}}$$

$$A := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -0.5 - 0.866i & -0.5 + 0.866i \\ 1 & -0.5 + 0.866i & -0.5 - 0.866i \end{pmatrix}$$

Three phase voltage or current from phase to sequence domain

```
symm3(ABC) := | R ← rows(ABC)
              | for i ∈ 0 .. R − 1
              |   | for j ∈ 0 .. 2
              |   |   M_j ← ABC_{i,j}
              |   | S ← A^(−1)·M
              |   | for j ∈ 0 .. 2
              |   |   PNZ_{i,j} ← S_j
              | return PNZ
```

)

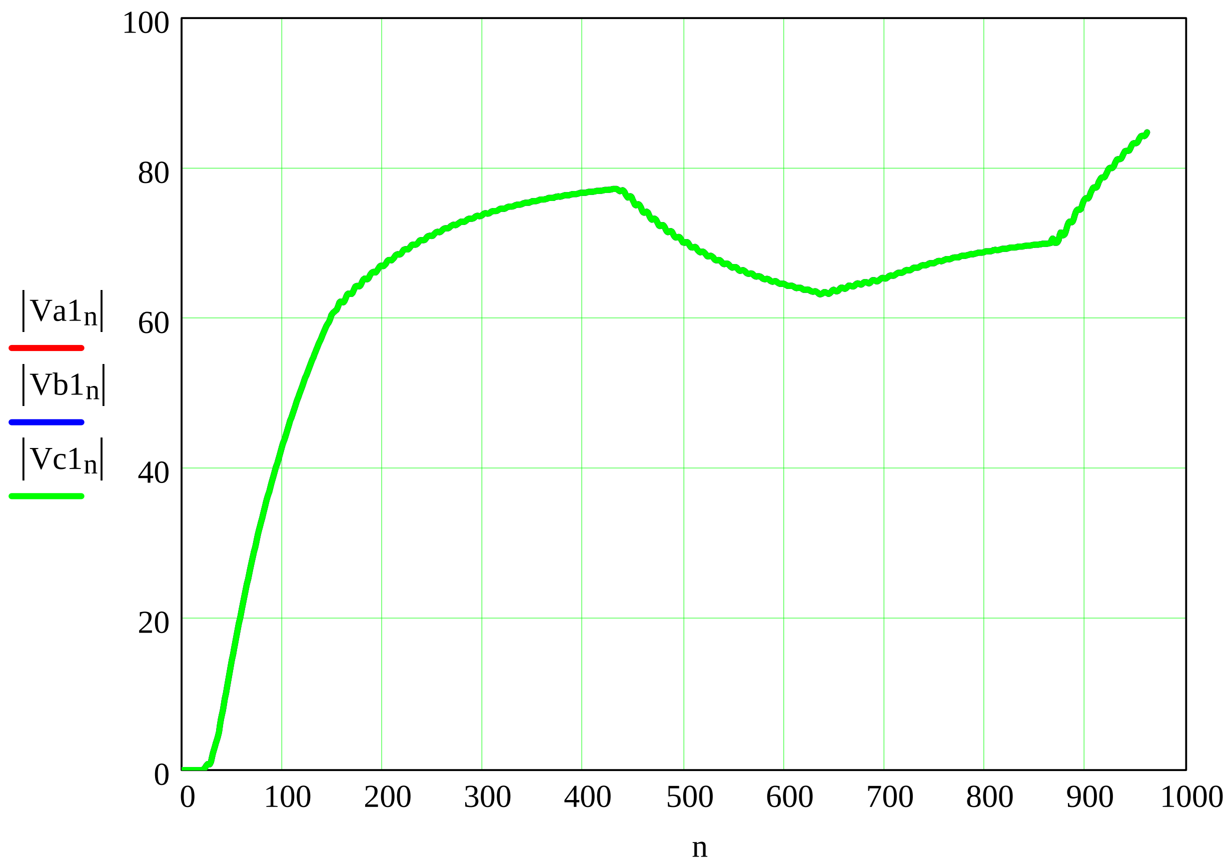
Compute Symmetrical Component impedance

$$ZLp := \begin{pmatrix} 4.420 + 50.971i & 2.35 + 25.968i & 2.266 + 24.446i \\ 2.35 + 25.968i & 4.57 + 50.796i & 2.35 + 25.968i \\ 2.266 + 24.446i & 2.35 + 25.968i & 4.402 + 50.971i \end{pmatrix}$$

$ZL012 := A^{-1} \cdot ZLp \cdot A \quad ZL1 := ZL012_{1,1} \; ZL0 := ZL012_{0,0}$

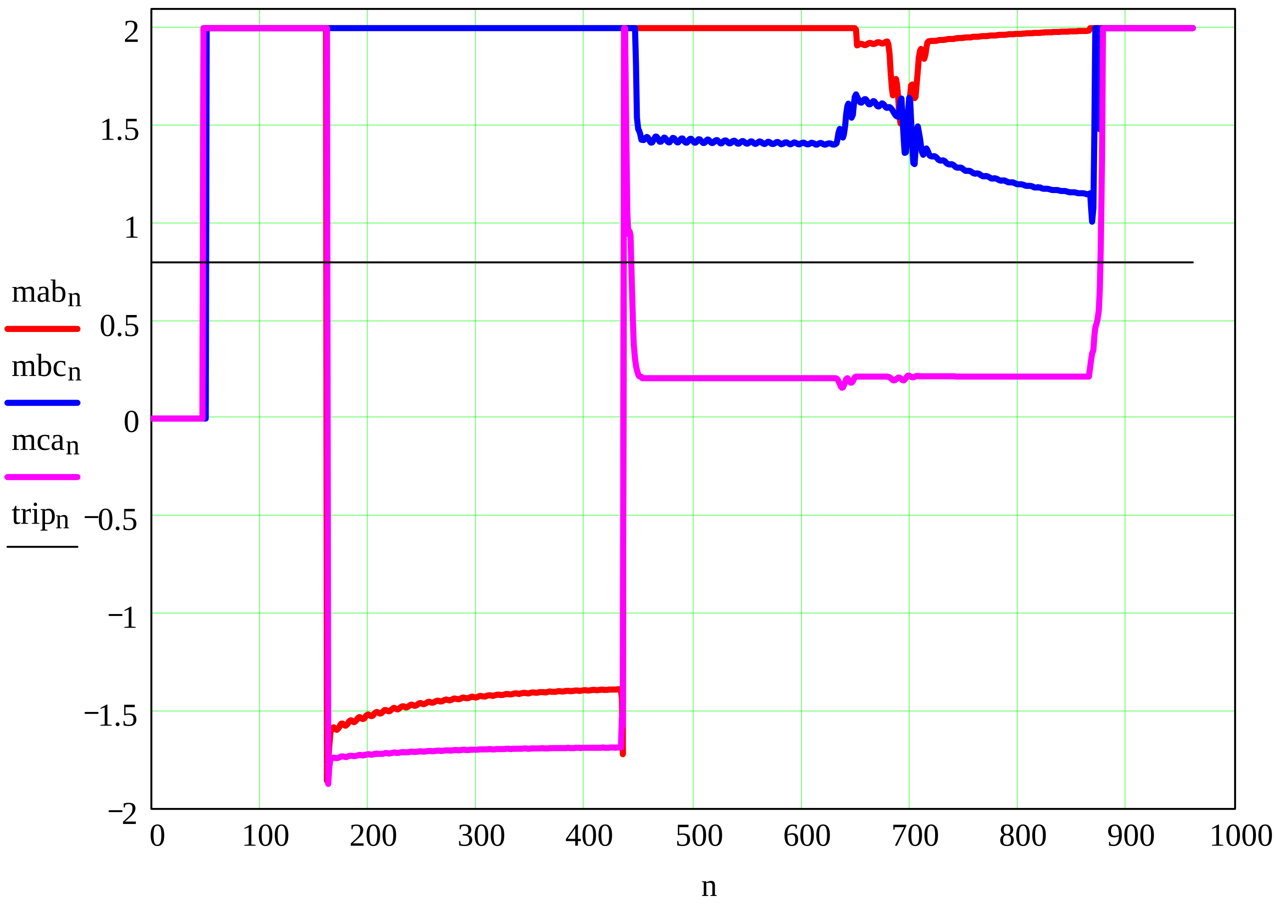
$$ZL012 = \begin{pmatrix} 9.108 + 101.834i & 0.353 - 0.297i & -0.425 - 0.152i \\ -0.425 - 0.152i & 2.142 + 25.452i & -0.923 + 0.537i \\ 0.353 - 0.297i & 0.935 + 0.537i & 2.142 + 25.452i \end{pmatrix}$$

Va1, Vb1, and Vc1



$mca_{700} = 0.217$ $mcg_{700} = 0.339$ $mag_{700} = 0.397$

Phase Distance Reach



Compute Phase Memory for Polarizing

$Vam := \mathrm{digitalfilter}(Va, Zmemn, Zmemd)$ $Vbm := \mathrm{digitalfilter}(Vb, Zmemn, Zmemd)$

$Vcm := \mathrm{digitalfilter}(Vc, Zmemn, Zmemd)$

$Vamr := \mathrm{digitalfilter}(Vam, Zn, Zd)$ $Vbmr := \mathrm{digitalfilter}(Vbm, Zn, Zd)$

$Vcmr := \mathrm{digitalfilter}(Vcm, Zn, Zd)$

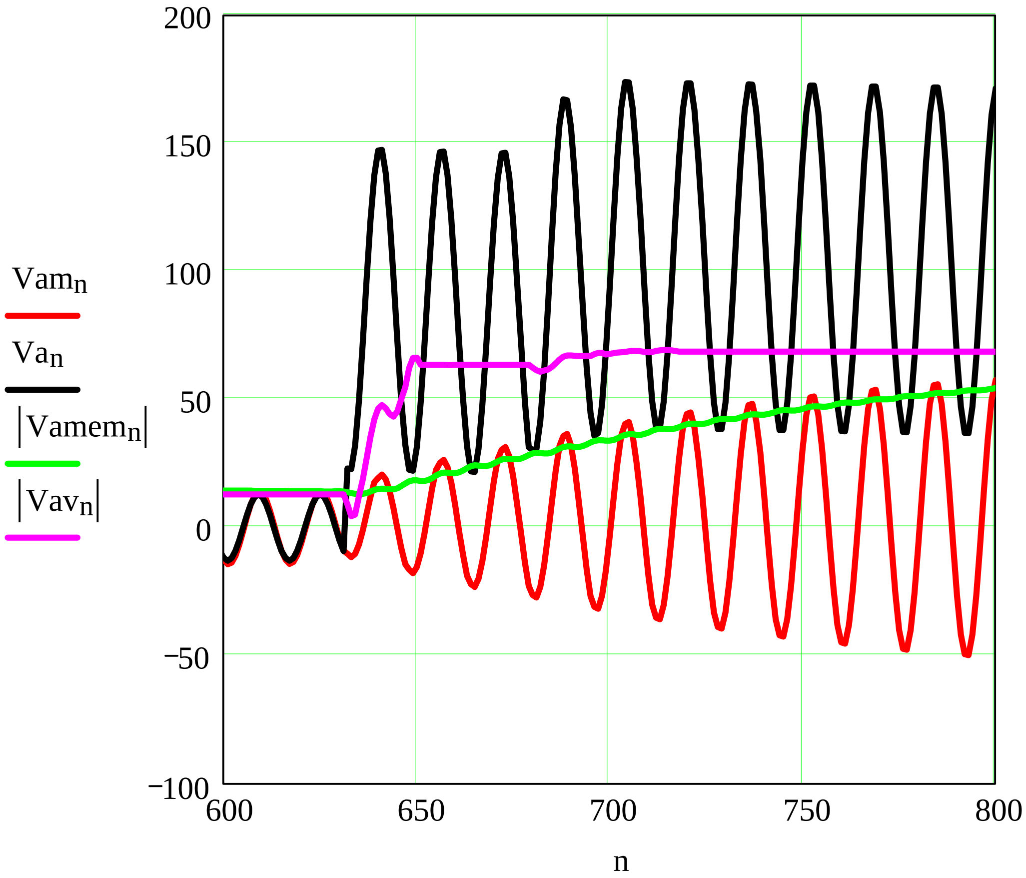
$Vami := \mathrm{digitalfilter}(Vamr, Znim, Zdim)$ $Vbmi := \mathrm{digitalfilter}(Vbmr, Znim, Zdim)$

$Vcmi := \mathrm{digitalfilter}(Vcmr, Znim, Zdim)$

$Vamem_n := Vamr_n + Vami_n i$ $Vbmem_n := Vbmr_n + Vbmi_n i$

$Vcmem_n := Vcmr_n + Vcmi_n i$

Phase A Response with Phase A Memory Filter Output Response



Phase A, B, C Memory Filter Response