This program computes the directional torque equations for relays Rs and Rr with various faults located at F1 and F2.

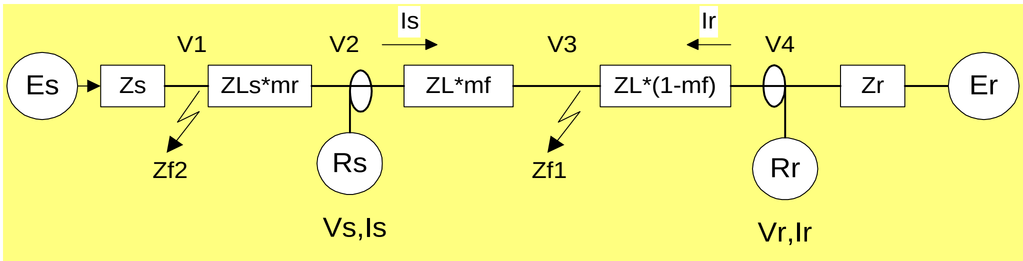

Figure 1. System single line diagram.

1.1. Transmission line calculations

1.1.2 Unbalanced line parameter matrix

$Line\_Length := 25$

$$ZL := \begin{pmatrix} 0.2201 + 0.9608i & 0.1175 + 0.4895i & 0.1133 + 0.4608i \\ 0.1175 + 0.4895i & 0.2285 + 0.9575i & 0.1175 + 0.4895i \\ 0.1133 + 0.4608i & 0.1175 + 0.4895i & 0.2201 + 0.9608i \end{pmatrix} \cdot Line\_Len$$

1.1.3 Balanced line parameter computations

$$Zm := \frac{\left(ZL_{0,1} + ZL_{0,2} + ZL_{2,1}\right)}{3} \quad Zs := \frac{\left(ZL_{0,0} + ZL_{1,1} + ZL_{2,2}\right)}{3}$$

$Zm = 2.902 + 11.998i$ $Zs = 5.573 + 23.992i$

1.1.3.1 Symmetrical component impedance calculations

$Z1 := (Zs - Zm)$ $Z1 = 2.67 + 11.994i$

$Z0 := (Zs + 2 \cdot Zm)$ $Z0 = 11.378 + 47.989i$

1.1.3.2 Asymmetrical compensation factor

$$k0 := \frac{(Z0 - Z1)}{3 \cdot Z1}$$

$k0 = 1.004 - 0.018i$

1.2 Network impedance matrices

1.2.1 Default R and S voltage source impedance

$$Zss := \begin{pmatrix} 1+6i & 0 & 0 \\ 0 & 1+6i & 0 \\ 0 & 0 & 1+6i \end{pmatrix}$$

$Zrr := Zss$

1.2.2 Default fault impedance matrices

$$Zff := \begin{pmatrix} 10^8 & 0 & 0 \\ 0 & 10^8 & 0 \\ 0 & 0 & 10^8 \end{pmatrix}$$

$Zfr := Zff$

1.2.3 Infinite impedance matrices

$$Ynull := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This matrix is used as a constant for building the system impedance matrix.

1.3 Voltage sources

1.3.1 Generator constants

Voltage Generator Voltage

Power system angle

Specify power angle in degress - positive for leading and negative for lagging,

VT and CT Ratios

$Vg := \frac{120}{\sqrt{3}}$

$ang := -1$

$VTR := 1$ $CTR := 1$

120 deg. shift operator

$$a := \cos\left(2\cdot\frac{\pi}{3}\right) + \sin\left(2\cdot\frac{\pi}{3}\right)i$$

Power angle shift operator

$$\delta := \cos\left(\frac{\pi\cdot ang}{180}\right) + \sin\left(\frac{\pi\cdot ang}{180}\right)i$$

1.3.2 S end voltage source

$$Es := \begin{pmatrix} Vg \\ Vg\cdot a^2 \\ Vg\cdot a \end{pmatrix}$$

1.3.3 R end voltage source

$$Er := \begin{pmatrix} Vg\cdot\delta \\ Vg\cdot\delta\cdot a^2 \\ Vg\cdot\delta\cdot a \end{pmatrix}$$

Zsystem Definitions

Output:
Z: System impedance matrix
For this system - the result is a [12X12] matrix of complex variables

Input:
Zs: S end voltage source Impedance matrix[1]
Zr: R end voltage source Impedance matrix[1]
ZL: Total length line impedance matrix[1]
Zff: Forward fault impedance matrix[1]
Zfr: Reverse fault impedance matrix[1]
mf: Percent of line length to forward[2]
mr: Percent of line length to reverse fault[2]

Notes:
1. [3X3] matrix of complex values
2. Scalar in range 0.0001 to 0.9999

1.4 Subroutines

1.4.1 System Impedance matrix

$$
\mathrm{Zsystem}(Zs, Zr, ZL, Zff, Zfr, mf, mr) :=
\begin{array}{|l}
Y11 \leftarrow Zs^{-1} + Zfr^{-1} + (mr \cdot ZL)^{-1} \\
Y12 \leftarrow -(mr \cdot ZL)^{-1} \\
Y13 \leftarrow Ynull \\
Y14 \leftarrow Ynull \\
Y21 \leftarrow Y12 \\
Y22 \leftarrow \left[(mf \cdot ZL)^{-1} + (mr \cdot ZL)^{-1} \right] \\
Y23 \leftarrow -(mf \cdot ZL)^{-1} \\
Y24 \leftarrow Ynull \\
Y31 \leftarrow Y13 \\
Y32 \leftarrow Y23 \\
Y33 \leftarrow \left[(mf \cdot ZL)^{-1} + [(1 - mf) \cdot ZL]^{-1} + Zff^{-1} \right] \\
Y34 \leftarrow -[(1 - mf) \cdot ZL]^{-1} \\
Y41 \leftarrow Y14 \\
Y42 \leftarrow Y24 \\
Y43 \leftarrow Y34 \\
Y44 \leftarrow \left[[(1 - mf) \cdot ZL]^{-1} + Zr^{-1} \right] \\
Y1 \leftarrow \mathrm{stack}(Y11, Y21) \\
Y1 \leftarrow \mathrm{stack}(Y1, Y31) \\
Y1 \leftarrow \mathrm{stack}(Y1, Y41) \\
Y2 \leftarrow \mathrm{stack}(Y12, Y22) \\
Y2 \leftarrow \mathrm{stack}(Y2, Y32) \\
Y2 \leftarrow \mathrm{stack}(Y2, Y42) \\
Y3 \leftarrow \mathrm{stack}(Y13, Y23) \\
Y3 \leftarrow \mathrm{stack}(Y3, Y33) \\
Y3 \leftarrow \mathrm{stack}(Y3, Y43) \\
Y4 \leftarrow \mathrm{stack}(Y14, Y24) \\
Y4 \leftarrow \mathrm{stack}(Y4, Y34) \\
Y4 \leftarrow \mathrm{stack}(Y4, Y44) \\
Y \leftarrow \mathrm{augment}(Y1, Y2) \\
Y \leftarrow \mathrm{augment}(Y, Y3) \\
Y \leftarrow \mathrm{augment}(Y, Y4) \\
\mathrm{return}\ \ Y^{-1}
\end{array}
$$

1.4.2 Compute system node voltages

Solve_Node_Voltages Definitions

Output:

Vn: Vnode voltages[1]

Input:

Es: S end voltage source[2]

Er: R end voltage source[2]

Zs: S end voltage source Impedance matrix[3]

Zr: R end voltage source Impedance matrix[3]

ZL: Total length line impedance matrix[3]

Zff: Forward fault impedance matrix[3]

Zfr: Reverse fault impedance matrix[3]

mf: Percent of line length to forward[4]

mr: Percent of line length to reverse fault[4]

Notes:

1. [1X12] column vector = $[V1 : V2 : V3 : V4]^T$
2. [1X3] column vector
3. [3X3] matrix of complex values
4. Scalar in range 0.0001 to 0.9999

$$\text{Solve\_Node\_Voltages}(Es, Er, Zs, Zr, ZL, Zff, Zfr, mf, mr) := \left| \begin{array}{l} Z \leftarrow \text{Zsystem}(Zs, Zr, ZL, Zff, Zfr, mf, mr) \\ \text{NullSrc} \leftarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ \text{Igen} \leftarrow Zs^{-1} \cdot Es \\ \text{Igen} \leftarrow \text{stack}(\text{Igen}, \text{NullSrc}) \\ \text{Igen} \leftarrow \text{stack}(\text{Igen}, \text{NullSrc}) \\ \text{Igen} \leftarrow \text{stack}\left[\text{Igen}, \left(Zr^{-1} \cdot Er\right)\right] \\ V \leftarrow Z \cdot \text{Igen} \\ \text{return } V \end{array} \right.$$

1.4.3 Phase to symmetrical component conversion matrix

h = scale vactor

$$h := 3$$

$$\text{Asys} := \left(\frac{1}{h}\right) \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

a = 120 degree phase shift operator

a^2 = 240 degree phase shift operator

1.4.4 Compute relay currents

Solve_currents Definitions

Output:

Ib: Branch currents[1]

Is - Send relay current
Ir - R end relay currents
Iflt - F1 fault current

Input:

V: Node voltage vector computed by Solve_Node_Voltages[2]

ZL: Total length line impedance matrix[3]

mf: Percent of line length to forward[4]

Notes:

1. 1X9 Column vector = $[Is : Ir : Iflt]^T$
2. [1X12] column vector = $[V1 : V2 : V3 : V4]^T$
3. 3X3 matrix of complex values
4. Scalar in range 0.0001 to 0.9999

$$
\text{Solve\_currents}(V, ZL, mf) := \left|
\begin{array}{l}
Is \leftarrow (mf \cdot ZL)^{-1} \cdot (\text{submatrix}(V,3,5,0,0) - \text{submatrix}(V,6,8,0,0)) \\
Ir \leftarrow [(1 - mf) \cdot ZL]^{-1} \cdot (\text{submatrix}(V,9,11,0,0) - \text{submatrix}(V,6,8,0,0)) \\
Iflt \leftarrow Is + Ir \\
Ib \leftarrow Is \\
Ib \leftarrow \text{stack}(Ib, Ir) \\
Ib \leftarrow \text{stack}(Ib, Iflt) \\
\text{return } Ib
\end{array}
\right.
$$

1.4.4 Rectangular to polar conversion

$$
\text{rect2polar}(R) := \left|
\begin{array}{l}
\text{for } i \in 0..2 \\
\quad \left|
\begin{array}{l}
m \leftarrow |R_i| \\
\alpha \leftarrow \arg(R_i) \cdot \dfrac{180}{\pi} \\
p \leftarrow (m \quad \alpha) \\
Pol \leftarrow p \quad \text{if } i < 1 \\
Pol \leftarrow \text{stack}(Pol, p) \quad \text{if } i > 0
\end{array}
\right. \\
\text{return } Pol
\end{array}
\right.
$$

Rectangular to polar coordinate conversion

Output: [V or I : Angle][1]

Input: [V or I complex variables][2]

Notes:

1. 3X2 matrix of scalar values
2. [3X1] vector of complex variables

1.5 Calculate prefault voltages and currents

The first calculation is for an unfaulted system. The locations of the forward (mf) and reverse (mr) faults doesn't matter at this point because the fault matrix represents essentually and open circuit. "Vprevault" is a matrix of node voltages for the prefault condition "Ibr" extracts the three phase Relay S and R currents as well as the total three phase forward fault current. "VSabc", "VRabc", and "VFabc" are 1x3 column vectors for the voltages seen br Relay s, Relay R and at the forward fault.

Location of forward fault $mf := 0.5$

Location of reverse fault $mr := 0.1$

Compute node voltages

$Vprefault := Solve\_Node\_Voltages(Es, Er, Zss, Zrr, ZL, Zff, Zfr, mf, mr)$

Extract Relay voltages

$VSabc\_prefault := submatrix(Vprefault, 3, 5, 0, 0)$

$VRabc\_prefault := submatrix(Vprefault, 6, 8, 0, 0)$

Display Relay and fault voltages in polar notation

$$rect2polar(VSabc\_prefault) = \begin{pmatrix} 69.291 & -0.282 \\ 69.288 & -120.288 \\ 69.281 & 119.716 \end{pmatrix}$$

$$rect2polar(VRabc\_prefault) = \begin{pmatrix} 69.278 & -0.524 \\ 69.279 & -120.524 \\ 69.279 & 119.476 \end{pmatrix}$$

Extract Relay and fault currents

$Ibr := Solve\_currents(Vprefault, ZL, mf)$

$ISprefault := submatrix(Ibr, 0, 2, 0, 0)$

$IRprefault := submatrix(Ibr, 3, 5, 0, 0)$

$IFLTprefault := submatrix(Ibr, 6, 8, 0, 0)$

Display Relay and fault currents in polar notation

$$rect2polar(ISprefault) = \begin{pmatrix} 0.047 & 11.713 \\ 0.048 & -109.117 \\ 0.047 & 129.193 \end{pmatrix}$$

$$rect2polar(IRprefault) = \begin{pmatrix} 0.047 & -168.286 \\ 0.048 & 70.883 \\ 0.047 & -50.807 \end{pmatrix}$$

$$rect2polar(IFLTprefault) = \begin{pmatrix} 6.928 \times 10^{-7} & -0.524 \\ 6.928 \times 10^{-7} & -120.524 \\ 6.928 \times 10^{-7} & 119.476 \end{pmatrix}$$

1.6 Calculate prefault voltages and currents

The second calculation is for an faulted system. The locations of the forward is establisted by setting the value of mf as a decimal fraction of the total line length. If mf equalt 0.25, then the fault is located 0.25 * the line length from Relay S toward Relay R. Line impedance is added to the source impedance by adjusting the value of mr shown as ZLs in Figure 1. The amount of impedance added is determined by Zls = mr*ZL where ZL is the system line impedance. This allows faults to be placed behind Relay S but still forward of relay R. Use small nonzero positive values (0.0001) for mr to add no additional impedance to the source impedance.

$mf := 0.1$ Forward fault location

$mr := 0.0001$ Reverse fault location

The parameters associated with the forward and reverse fault matrix are illustrated in Figure 2. Use impedance values of 10^6 for open circuits. Phase to phase faults are generated by setting Zab, Zbc, and / or Zac to small values. Phase to ground faults are generated by setting Zaa, Zbb, and / or Zcc to small values. The minimum fault impedance should be limited to 0.0001. Fault impedances can be complex. Not specifying a fault impedance defaults to an open circuit fault matrix. Chaning the left hand variable fro Zff to Zfr moves the fault matrix to the Zf2 position making the fault matrix behind Relay S.

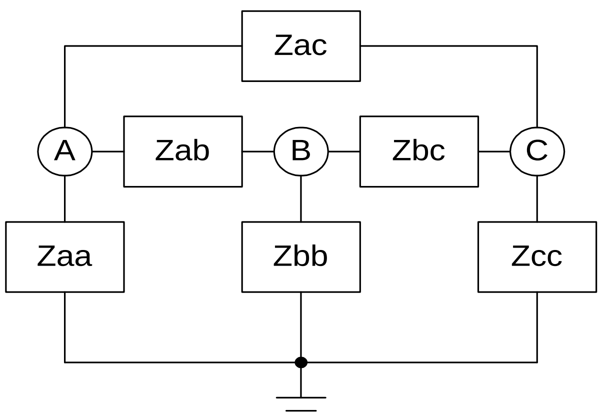

Figure 2. Fault matrix diagram.

$Zaa := 0.01$ $Zbb := 10^6$ $Zcc := 10^6$ $Zab := 10^6$ $Zbc := 10^6$ $Zac := 10^6$

$$Zff := \begin{bmatrix} \left(Zaa^{-1} + Zab^{-1} + Zac^{-1}\right) & -Zab^{-1} & -Zac^{-1} \\ -Zab^{-1} & \left(Zbb^{-1} + Zab^{-1} + Zbc^{-1}\right) & -Zbc^{-1} \\ -Zac^{-1} & -Zbc^{-1} & \left(Zcc^{-1} + Zac^{-1} + Zbc^{-1}\right) \end{bmatrix}^{-1}$$

1.6.1 Compute relay voltages and currents for the faulted system

Location of forward fault $mf := 0.05$ Reverse fault location $mr := 0.05$

Compute node voltages

$Vnode := Solve\_Node\_Voltages(Es, Er, Zss, Zrr, ZL, Zff, Zfr, mf, mr)$

Extract Relay and fault voltages

$VSabc := submatrix(Vnode, 3, 5, 0, 0)$

$VRabc := submatrix(Vnode, 9, 11, 0, 0)$

$VFabc := submatrix(Vnode, 6, 8, 0, 0)$

Display Relay and fault voltages in polar notation

$$rect2polar(VSabc) = \begin{pmatrix} 10.378 & -3.021 \\ 73.765 & -126.066 \\ 73.727 & 124.941 \end{pmatrix}$$

$$rect2polar(VRabc) = \begin{pmatrix} 53.006 & -1.682 \\ 67.848 & -118.844 \\ 68.188 & 117.48 \end{pmatrix}$$

$$rect2polar(VFabc) = \begin{pmatrix} 0.107 & -79.358 \\ 77.091 & -129.716 \\ 77.249 & 128.122 \end{pmatrix}$$

1.6.2 Extract Relay and fault currents

$Ibr := Solve\_currents(Vnode, ZL, mf)$

$ISabc := submatrix(Ibr, 0, 2, 0, 0)$

$IRabc := submatrix(Ibr, 3, 5, 0, 0)$

$IFLTabc := submatrix(Ibr, 6, 8, 0, 0)$

Display Relay and fault currents in polar notation

$$rect2polar(ISabc) = \begin{pmatrix} 7.987 & -79.371 \\ 0.485 & -81.406 \\ 0.349 & -83.264 \end{pmatrix}$$

$$rect2polar(IRabc) = \begin{pmatrix} 2.678 & -79.318 \\ 0.485 & 98.61 \\ 0.35 & 96.747 \end{pmatrix}$$

$$rect2polar(IFLTabc) = \begin{pmatrix} 10.666 & -79.358 \\ 2.587 \times 10^{-4} & -112.76 \\ 2.593 \times 10^{-4} & 111.214 \end{pmatrix}$$

1.6.3 Set maximum torque angle to the argumant of the positive sequence impedance

$MTA := arg(Z1)$

1.6.4 Compute cross polarizing directional torque values

1.6.4.1 Compute phase to phase voltages at Relay S and R

$VSab := VSabc_0 - VSabc_1$

$VSbc := VSabc_1 - VSabc_2$

$VSca := VSabc_2 - VSabc_0$

$VRab := VRabc_0 - VRabc_1$

$VRbc := VRabc_1 - VRabc_2$

$VRca := VRabc_2 - VRabc_0$

1.6.4.1 Compute torque values Relay S and R

$TSa := |VSbc| \cdot |ISabc_0| \cdot \cos\left[\arg(VSbci) - \left(\arg\left(ISabc_0\right) + MTA\right)\right]$ $TSa = 958.78$

$TSb := |VSca| \cdot |ISabc_1| \cdot \cos\left[\arg(VScai) - \left(\arg\left(ISabc_1\right) + MTA\right)\right]$ $TSb = -27.774$

$TSc := |VSab| \cdot |ISabc_2| \cdot \cos\left[\arg(VSabi) - \left(\arg\left(ISabc_2\right) + MTA\right)\right]$ $TSc = -22.433$

$TRa := |VRbc| \cdot |IRabc_0| \cdot \cos\left[\arg(VRbci) - \left(\arg\left(IRabc_0\right) + MTA\right)\right]$ $TRa = 321.142$

$TRb := |VRca| \cdot |IRabc_1| \cdot \cos\left[\arg(VRcai) - \left(\arg\left(IRabc_1\right) + MTA\right)\right]$ $TRb = 27.223$

$TRc := |VRab| \cdot |IRabc_2| \cdot \cos\left[\arg(VRabi) - \left(\arg\left(IRabc_2\right) + MTA\right)\right]$ $TRc = 23.155$

1.6.5 Compute symmetrical component polarizing torque directional values

1.6.5.1 Compute volatage and current symmetrical components at Relay S and R

$VS012 := Asys \cdot VSabc$

$VR012 := Asys \cdot VRabc$

$IS012 := Asys \cdot ISabc$

$IR012 := Asys \cdot IRabc$

$\arg(Z1) = 77.45\,deg$

$$rect2polar(VS012) = \begin{pmatrix} 25.097 & 179.8 \\ 52.394 & -0.726 \\ 16.93 & 179.9 \end{pmatrix}$$

$$rect2polar(VR012) = \begin{pmatrix} 3.741 & -177.491 \\ 62.989 & -0.966 \\ 6.268 & -176.979 \end{pmatrix}$$

$$rect2polar(IS012) = \begin{pmatrix} 2.94 & -79.637 \\ 2.522 & -78.32 \\ 2.526 & -80.111 \end{pmatrix}$$

$$rect2polar(IR012) = \begin{pmatrix} 0.615 & -78.029 \\ 1.035 & -81.88 \\ 1.03 & -77.516 \end{pmatrix}$$

1.6.5.2 Compute zero sequence current polarized torque values Relay S and R

The torque equation is genarically - T = |Ipol| |Iphase| cos(angle(Ipol) - angle(Iphase)). For line to ground faults, the zero sequence current is in phase with the faulted phase. This works for two line to singel and two line to ground faults but not for balanced three phase to groung faults nor for phase to phase faults since these faults do not normaly generatezero sequence currents.

$$TI0Sa := \left|IS012_0\right| \cdot \left|ISabc_0\right| \cdot \cos\left[\arg\left(IS012_0\right) - \left(\arg\left(ISabc_0\right) + 0\right)\right]$$ $TI0Sa = 23.486$

$$TI0Sb := \left|IS012_0\right| \cdot \left|ISabc_1\right| \cdot \cos\left[\arg\left(IS012_0\right) - \left(\arg\left(ISabc_1\right) + 0\right)\right]$$ $TI0Sb = 1.427$

$$TI0Sc := \left|IS012_0\right| \cdot \left|ISabc_2\right| \cdot \cos\left[\arg\left(IS012_0\right) - \left(\arg\left(ISabc_2\right) + 0\right)\right]$$ $TI0Sc = 1.025$

$$TI0Ra := \left|IR012_0\right| \cdot \left|IRabc_0\right| \cdot \cos\left[\arg\left(IR012_0\right) - \left(\arg\left(IRabc_0\right) + 0\right)\right]$$ $TI0Ra = 1.647$

$$TI0Rb := \left|IR012_0\right| \cdot \left|IRabc_1\right| \cdot \cos\left[\arg\left(IR012_0\right) - \left(\arg\left(IRabc_1\right) + 0\right)\right]$$ $TI0Rb = -0.298$

$$TI0Rc := \left|IR012_0\right| \cdot \left|IRabc_2\right| \cdot \cos\left[\arg\left(IR012_0\right) - \left(\arg\left(IRabc_2\right) + 0\right)\right]$$ $TI0Rc = -0.214$

1.6.5.3 Compute zero sequence voltage polarized torque values Relay S and R

The torque equation is genarically - T = -|Vpol| |Iphase| cos(angle(a^pVpol) - angle(Iphase)) where a is the 120 degree operator and p is 0 for phase a, 2 for phase b, and 1 for phase c. For line to ground faults, the zero sequence voltage is 180 degrees out of phase with the voltage of the faulted phase. This works for two line to singel and two line to ground faults but not for balanced three phase to groung faults nor for phase to phase faults.

$$TV0Sa := -\left|VS012_0\right| \cdot \left|ISabc_0\right| \cdot \cos\left[\arg\left(VS012_0\right) - \left(\arg\left(ISabc_0\right) + 0\right)\right]$$ $TV0Sa = 37.659$

$$TV0Sb := -\left|VS012_0\right| \cdot \left|ISabc_1\right| \cdot \cos\left[\arg\left(VS012_0\right) - \left(\arg\left(ISabc_1\right) + 0\right)\right]$$ $TV0Sb = 1.863$

$$TV0Sc := -\left|VS012_0\right| \cdot \left|ISabc_2\right| \cdot \cos\left[\arg\left(VS012_0\right) - \left(\arg\left(ISabc_2\right) + 0\right)\right]$$ $TV0Sc = 1.059$

$$TV0Ra := -\left|VR012_0\right| \cdot \left|IRabc_0\right| \cdot \cos\left[\arg\left(VR012_0\right) - \left(\arg\left(IRabc_0\right) + 0\right)\right]$$ $TV0Ra = 1.424$

$$TV0Rb := -\left|VR012_0\right| \cdot \left|IRabc_1\right| \cdot \cos\left[\arg\left(VR012_0\right) - \left(\arg\left(IRabc_1\right) + 0\right)\right]$$ $TV0Rb = -0.193$

$$TV0Rc := -\left|VR012_0\right| \cdot \left|IRabc_2\right| \cdot \cos\left[\arg\left(VR012_0\right) - \left(\arg\left(IRabc_2\right) + 0\right)\right]$$ $TV0Rc = -0.097$

1.6.5.4 Compute negative sequence current polarized torque values Relay S and R

The torque equation is genarically - T = -|Vpol| |Iphase| cos(angle(a^pVpol) - angle(Iphase)) where a is the 120 degree operator and p is 0 for phase a, 2 for phase b, and 1 for phase c. For line to ground faults, the zero sequence voltage is 180 degrees out of phase with the voltage of the faulted phase. This works for two line to singel and two line to ground faults but not for balanced three phase to groung faults nor for phase to phase faults.

$$TI2Sa := |IS012_2| \cdot |ISabc_0| \cdot \cos[\arg(IS012_2) - (\arg(ISabc_0) + 0)]$$ $TI2Sa = 20.171$

$$TI2Sb := |IS012_2| \cdot |ISabc_1| \cdot \cos\left[\arg(IS012_2) - \left(\arg\left(a^2 \cdot ISabc_1\right) + 0\right)\right]$$ $TI2Sb = -0.637$

$$TI2Sc := |IS012_2| \cdot |ISabc_2| \cdot \cos[\arg(IS012_2) - (\arg(a \cdot ISabc_2) + 0)]$$ $TI2Sc = -0.398$

$$TI2Ra := |IR012_2| \cdot |IRabc_0| \cdot \cos[\arg(IR012_2) - (\arg(IRabc_0))]$$ $TI2Ra = 2.758$

$$TI2Rb := |IR012_2| \cdot |IRabc_1| \cdot \cos[\arg(IR012_2) - (\arg(a \cdot IRabc_1))]$$ $TI2Rb = 0.22$

$$TI2Rc := |IR012_2| \cdot |IRabc_2| \cdot \cos\left[\arg(IR012_2) - \left(\arg\left(a^2 \cdot IRabc_2\right)\right)\right]$$ $TI2Rc = 0.21$

1.6.5.4 Compute negative sequence voltage polarized torque values Relay S and R

The torque equation is genarically - T = -|Vpol| |Iphase| cos(angle(a^pVpol) - angle(Iphase)) where a is the 120 degree operator and p is 0 for phase a, 2 for phase b, and 1 for phase c. For line to ground faults, the zero sequence voltage is 180 degrees out of phase with the voltage of the faulted phase. This works for two line to singel and two line to ground faults but not for balanced three phase to groung faults nor for phase to phase faults.

$$TV2Sa := -|VS012_2| \cdot |ISabc_0| \cdot \cos[\arg(VS012_2) - (\arg(ISabc_0) + MTA)]$$ $TV2Sa = 135.155$

$$TV2Sb := -|VS012_2| \cdot |ISabc_1| \cdot \cos[\arg[(a \cdot VS012)_2] - (\arg(ISabc_1) + MTA)]$$ $TV2Sb = -4.579$

$$TV2Sc := -|VS012_2| \cdot |ISabc_2| \cdot \cos\left[\arg\left(a^2 \cdot VS012_2\right) - (\arg(ISabc_2) + MTA)\right]$$ $TV2Sc = -2.432$

$$TV2Ra := -|VR012_2| \cdot |IRabc_0| \cdot \cos[\arg(VR012_2) - (\arg(IRabc_0) + MTA)]$$ $TV2Ra = 16.726$

$$TV2Rb := -|VR012_2| \cdot |IRabc_1| \cdot \cos[\arg(a \cdot VR012_2) - (\arg(IRabc_1) + MTA)]$$ $TV2Rb = 1.829$

$$TV2Rc := -|VR012_2| \cdot |IRabc_2| \cdot \cos\left[\arg\left(a^2 \cdot VR012_2\right) - (\arg(IRabc_2) + MTA)\right]$$ $TV2Rc = 0.791$

Reference equations - 6026 "New Ground Direction Elements Operate Reliably for Changing System Conditions", A. Guzman, J. Roberts, and D. Hou. These algorithms should be used for ground faults only.

Relay S zero sequence voltage polarized torque value

$$T32SV0 := \frac{-\left(\mathrm{Re}\left(VS012_0 \cdot \overline{\left[\left(\frac{Z0}{|Z0|}\right) \cdot IS012_0\right]}\right)\right)}{\left(\left|IS012_0\right|\right)^2}$$

$T32SV0 = 8.525$

Relay R zero sequence voltage polarized torque value

$$T32RV0 := \frac{-\left[\mathrm{Re}\left[\left(VR012_0\right) \cdot \overline{\left[\left(\frac{Z0}{|Z0|}\right) \cdot IR012_0\right]}\right]\right]}{\left(\left|IR012_0\right|\right)^2}$$

$T32RV0 = 6.069$

Relay S negative sequence voltage polarized torque value

$$T32SV2 := \frac{-\left(\mathrm{Re}\left(VS012_2 \cdot \overline{\left[\left(\frac{Z1}{|Z1|}\right) \cdot IS012_2\right]}\right)\right)}{\left(\left|IS012_2\right|\right)^2}$$

$T32SV2 = 6.697$

Relay R negative sequence voltage polarized torque value

$$T32RV2 := \frac{-\left[\mathrm{Re}\left[\left(VR012_2\right) \cdot \overline{\left[\left(\frac{Z1}{|Z1|}\right) \cdot IR012_2\right]}\right]\right]}{\left(\left|IR012_2\right|\right)^2}$$

$T32RV2 = 6.074$

$$r := 0.8$$

$$VS\_012 := Asys \cdot VSabc\_prefault$$

$$rect2polar(VS\_012) = \begin{pmatrix} 9.376 \times 10^{-4} & -42.389 \\ 69.287 & -0.285 \\ 5.09 \times 10^{-3} & 42.153 \end{pmatrix}$$

$$VS1pol := VS\_012_1$$

$$VR\_012 := Asys \cdot VRabc\_prefault$$

$$rect2polar(VR\_012) = \begin{pmatrix} 1.042 \times 10^{-4} & 137.604 \\ 69.279 & -0.524 \\ 5.657 \times 10^{-4} & -137.849 \end{pmatrix}$$

$$VR1pol := VR\_012_1$$

$$TSAG := \left[r \cdot Z1 \cdot \left(ISabc_0 + 3 \cdot k0 \cdot IS012_0\right) - VSabc_0\right] \cdot \overline{\left(a^0 \cdot VS1pol\right)}$$ $$Re(TSAG) = 1.075 \times 10^4$$

$$TSBG := \left[r \cdot Z1 \cdot \left(ISabc_1 + 3 \cdot k0 \cdot IS012_0\right) - VSabc_1\right] \cdot \overline{\left(a^2 \cdot VS1pol\right)}$$ $$Re(TSBG) = -7.976 \times 10^3$$

$$TSCG := \left[r \cdot Z1 \cdot \left(ISabc_2 + 3 \cdot k0 \cdot IS012_0\right) - VSabc_2\right] \cdot \overline{\left(a^1 \cdot VS1pol\right)}$$ $$Re(TSCG) = -8.508 \times 10^3$$

$$TSAB := \left[r \cdot Z1 \cdot \left(ISabc_0 - ISabc_1\right) - \left(VSabc_0 - VSabc_1\right)\right] \cdot \overline{\left[\left(-a^1\right) \cdot VS1poli\right]}$$ $$Re(TSAB) = -909.357$$

$$TSBC := \left[r \cdot Z1 \cdot \left(ISabc_1 - ISabc_2\right) - \left(VSabc_1 - VSabc_2\right)\right] \cdot \overline{\left[\left(-a^0\right) \cdot VS1poli\right]}$$ $$Re(TSBC) = -8.321 \times 10^3$$

$$TSCA := \left[r \cdot Z1 \cdot \left(ISabc_2 - ISabc_0\right) - \left(VSabc_2 - VSabc_0\right)\right] \cdot \overline{\left[\left(-a^2\right) \cdot VS1poli\right]}$$ $$Re(TSCA) = -706.715$$

$$TRAG := \left[r \cdot Z1 \cdot \left(IRabc_0 + 3 \cdot k0 \cdot IR012_0\right) - VRabc_0\right] \cdot \overline{\left(a^0 \cdot VR1pol\right)}$$ $$Re(TRAG) = -585.756$$

$$TRBG := \left[r \cdot Z1 \cdot \left(IRabc_1 + 3 \cdot k0 \cdot IR012_0\right) - VRabc_1\right] \cdot \overline{\left(a^2 \cdot VR1pol\right)}$$ $$Re(TRBG) = -5.161 \times 10^3$$

$$TRCG := \left[r \cdot Z1 \cdot \left(IRabc_2 + 3 \cdot k0 \cdot IR012_0\right) - VRabc_2\right] \cdot \overline{\left(a^1 \cdot VR1pol\right)}$$ $$Re(TRCG) = -5.236 \times 10^3$$

$$TRAB := \left[r \cdot Z1 \cdot \left(IRabc_0 - IRabc_1\right) - \left(VRabc_0 - VRabc_1\right)\right] \cdot \overline{\left[\left(-a^1\right) \cdot VR1poli\right]}$$ $$Re(TRAB) = -5.309 \times 10^3$$

$$TRBC := \left[r \cdot Z1 \cdot \left(IRabc_1 - IRabc_2\right) - \left(VRabc_1 - VRabc_2\right)\right] \cdot \overline{\left[\left(-a^0\right) \cdot VR1poli\right]}$$ $$Re(TRBC) = -8.306 \times 10^3$$

$$TRCA := \left[r \cdot Z1 \cdot \left(IRabc_2 - IRabc_0\right) - \left(VRabc_2 - VRabc_0\right)\right] \cdot \overline{\left[\left(-a^2\right) \cdot VR1poli\right]}$$ $$Re(TRCA) = -5.406 \times 10^3$$

mf=0.99						mr=0.05					
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z2	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	CA
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z2	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	CA
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z2	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	CA
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z1	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z1	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA

mf=0.99						mr=0.05					
AG Z2	BG Z2	CG Z2	AB Z3	BC Z3	CA Z3	AG Z3			AB Z3	BC Z3	C
AG Z2	BG Z2	CG Z2	AB Z3	BC Z3	CA Z3		BG Z3		AB Z3	BC Z3	C
AG Z2	BG Z2	CG Z2	AB Z3	BC Z3	CA Z3			CG Z3	AB Z3	BC Z3	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2	AG Z2			AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2		BG Z2		AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2			CG Z2	AB Z2	BC Z2	C

mf=0.99						mr=0.05					
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z1	BC Z1	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C

mf=0.99						mr=0.05				
AG Z2	BG Z2	CG Z2	AB Z2	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3
AG Z2	BG Z2	CG Z2	AB Z2	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3
AG Z2	BG Z2	CG Z2	AB Z2	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3
AG Z1	BG Z1	CG Z1	AB Z1	BC Z2	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2
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AG Z1	BG Z1	CG Z1	AB Z1	BC Z2	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2

mf=0.99						mr=0.05					
AG Z1	BG Z3	CG Z2	AB Z3	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z1	BG Z3	CG Z2	AB Z3	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z1	BG Z3	CG Z2	AB Z3	BC Z3	CA Z3	AG Z3	BG Z3	CG Z3	AB Z3	BC Z3	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C
AG Z1	BG Z1	CG Z1	AB Z2	BC Z2	CA Z2	AG Z2	BG Z2	CG Z2	AB Z2	BC Z2	C

mf=0.99

mr=0.05

AG Z2 BG Z2 CG Z2 AB Z2 BC Z2 CA Z2 AG Z1 BG Z3 CG Z2 AB Z3 BC Z3

AG Z1 BG Z1 CG Z1 AB Z1 BC Z1 CA Z1 AG Z2 BG Z2 CG Z2 AB Z2 BC Z2

mf=0.99

mr=0.05

AG Z2 BG Z2 CG Z2 AB Z2 BC Z2 CA Z2 AG Z1 BG Z3 CG Z2 AB Z3 BC Z3

AG Z1 BG Z1 CG Z1 AB Z1 BC Z1 CA Z1 AG Z2 BG Z2 CG Z2 AB Z2 BC Z2

CA Z3

CA Z3

CA Z3

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