EE421 Unbalanced Line Example

Define transformation

$$a := 1 \cdot e^{j \cdot \frac{2\cdot\pi}{3}}$$

$$A1 := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Define Constants:

$\rho := 100\text{ohm}\cdot\text{m}$ $\quad$ $\text{freq} := 60\text{Hz}$

$\mu o := 4\cdot\pi\cdot 10^{-7}\frac{\text{H}}{\text{m}}$ $\quad$ $\text{Icons} := \frac{\mu o}{2\cdot\pi}$

$\text{CarsonsResistConst} := 9.869 \times 10^{-7}\frac{\text{ohm}}{\text{m}\cdot\text{Hz}}$ $\quad$ $\text{De\_const} := 2160\cdot\frac{\text{ft}\cdot\text{Hz}^{0.5}}{(\text{ohm}\cdot\text{m})^{0.5}}$

Compute the series impedance matrix for the line configuration of Figure P4.4 where the conductor is 336,400 CM, 26/7 Strand ACSR. Assume line is 50 miles long

Define heights relative to the earth

$\text{Hgw} := 36\text{ft} + 11\text{ft}$

$\text{Ha} := \text{Hgw} - 6.5\text{ft}$ $\quad$ $\text{Ha} = 40.5\,\text{ft}$

$\text{Hb} := 36\text{ft}$

$\text{Hc} := \text{Ha} - 8\text{ft}$ $\quad$ $\text{Hc} = 32.5\,\text{ft}$

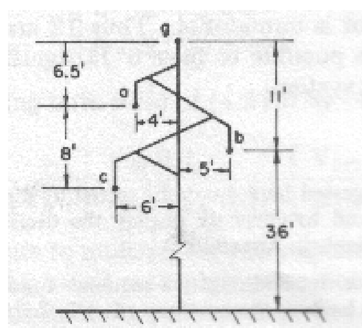

Fig. P4.4.

Define horizontal position relative to center of the tower (some are negative).

$\text{Xa} := -4\text{ft}$ $\quad$ $\text{Xb} := 5\text{ft}$ $\quad$ $\text{Xc} := -6\text{ft}$

Now calculate distance between conductors (be careful of negative signs)

$\text{Dab} := \sqrt{(\text{Ha}-\text{Hb})^2 + (\text{Xb}-\text{Xa})^2}$ $\quad$ $\text{Dab} = 10.062\,\text{ft}$

$$Dac := \sqrt{(Ha - Hc)^2 + (Xc - Xa)^2} \qquad Dac = 8.246\,ft$$

$$Dbc := \sqrt{(Hb - Hc)^2 + (Xb - Xc)^2} \qquad Dbc = 11.543\,ft$$

GMR and Rac from table B.8 (entry for Linnet in Appendix 8 of Bergen and Vittal)

$$GMR := 0.0244\,ft \qquad diameter := 0.721\,in$$

$$Rac := 0.273\,\frac{ohm}{mi}$$ at 25C and 60Hz

$$Rd := CarsonsResistConst \cdot freq$$

$$Rperlength := \begin{pmatrix} Rac + Rd & Rd & Rd \\ Rd & Rac + Rd & Rd \\ Rd & Rd & Rac + Rd \end{pmatrix} \qquad Rperlength = \begin{pmatrix} 0.368 & 0.095 & 0.095 \\ 0.095 & 0.368 & 0.095 \\ 0.095 & 0.095 & 0.368 \end{pmatrix} \frac{ohm}{mi}$$

$$De := De\_const \cdot \sqrt{\frac{\rho}{freq}} \qquad De = 2788.55\,ft$$

$$Lperlength := \begin{pmatrix} Icons \cdot \ln\left(\frac{De}{GMR}\right) & Icons \cdot \ln\left(\frac{De}{Dab}\right) & Icons \cdot \ln\left(\frac{De}{Dac}\right) \\ Icons \cdot \ln\left(\frac{De}{Dab}\right) & Icons \cdot \ln\left(\frac{De}{GMR}\right) & Icons \cdot \ln\left(\frac{De}{Dbc}\right) \\ Icons \cdot \ln\left(\frac{De}{Dac}\right) & Icons \cdot \ln\left(\frac{De}{Dbc}\right) & Icons \cdot \ln\left(\frac{De}{GMR}\right) \end{pmatrix}$$

$$Lperlength = \begin{pmatrix} 3.749 & 1.81 & 1.874 \\ 1.81 & 3.749 & 1.766 \\ 1.874 & 1.766 & 3.749 \end{pmatrix} \frac{mH}{mi}$$

$$Zabc\_perlength := Rperlength + j \cdot 2 \cdot \pi \cdot freq \cdot Lperlength \qquad Length := 50mi$$

$$Zabc := Zabc\_perlength \cdot Length$$

$$Zabc = \begin{pmatrix} 18.415 + 70.66i & 4.765 + 34.124i & 4.765 + 35.332i \\ 4.765 + 34.124i & 18.415 + 70.66i & 4.765 + 33.291i \\ 4.765 + 35.332i & 4.765 + 33.291i & 18.415 + 70.66i \end{pmatrix} \Omega$$

4.34 Consider the line configuration shown in Figure P4.4. Instead of using a single conductor of 336,400 CM ACSR in each phase, with current carrying capacity of 530 amperes, suppose that each phase consists of a two-conductor bundle of two 3/0 ACSR conductors with capacity of 300 amperes/conductor. Let the two conductors of each bundle be seperated by 1.0ft vertically.

(a) Compute the phase impedance matrix Zabc for the bundled conductor configuration and compare with the previous solution (problem 4.20).

Since 4.20 neglects the ground wire, it will be neglected here too.

$$\text{Rac3} := 0.560\,\frac{\text{ohm}}{\text{mi}}$$ from table B.8 at 25C and 60Hz

$\text{GMR3} := 0.006\text{ft}$ diameter3 := 0.502in

Spacing between conductors

Within the bundle:

$\text{Da1a2} := 1.0\text{ft}$ $\text{Db1b2} := 1.0\text{ft}$ $\text{Dc1c2} := 1.0\text{ft}$

Calculate geometric mean radius of the bundle and use the 3x3 matrix method. The is an approximation of the 6x6 matrix approach.

$$\text{Rsbundle} := \sqrt{\text{GMR3}\cdot\text{Da1a2}}$$

$$\text{Lperlength\_alt} := \text{Icons}\cdot\begin{pmatrix} \ln\left(\frac{\text{De}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{De}}{\text{Dab}}\right) & \ln\left(\frac{\text{De}}{\text{Dac}}\right) \\ \ln\left(\frac{\text{De}}{\text{Dab}}\right) & \ln\left(\frac{\text{De}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{De}}{\text{Dbc}}\right) \\ \ln\left(\frac{\text{De}}{\text{Dac}}\right) & \ln\left(\frac{\text{De}}{\text{Dbc}}\right) & \ln\left(\frac{\text{De}}{\text{Rsbundle}}\right) \end{pmatrix}$$

The resistance matrix must also be modified since there are now parallel conductors

$$\text{Rperlength\_bu\_new} := \begin{pmatrix} \frac{\text{Rac3}}{2} + \text{Rd} & \text{Rd} & \text{Rd} \\ \text{Rd} & \frac{\text{Rac3}}{2} + \text{Rd} & \text{Rd} \\ \text{Rd} & \text{Rd} & \frac{\text{Rac3}}{2} + \text{Rd} \end{pmatrix}$$

$$\text{Zabc\_bu} := (\text{Rperlength\_bu\_new} + j \cdot 2 \cdot \pi \cdot \text{freq} \cdot \text{Lperlength\_alt}) \cdot \text{Length}$$

$$\text{Zabc\_bu} = \begin{pmatrix} 18.7648 + 63.6514i & 4.7648 + 34.1242i & 4.7648 + 35.3318i \\ 4.7648 + 34.1242i & 18.7648 + 63.6514i & 4.7648 + 33.2911i \\ 4.7648 + 35.3318i & 4.7648 + 33.2911i & 18.7648 + 63.6514i \end{pmatrix} \text{ohm}$$

Observations:

(1) The resistance terms are all the same.
(2) The self inductance terms are noticably smaller, due to the larger effective radius of the phase conductors
(3) The mutual inductance terms are basically the same, since the distances between the phases is unchanged

(b) Compute the sequence impedance matrix for both the new and old conductor arrangements and compare.

$$\text{Z012\_p4\_20} := \text{A1}^{-1} \cdot \text{Zabc} \cdot \text{A1} \qquad \text{Z012\_p4\_34} := \text{A1}^{-1} \cdot \text{Zabc\_bu} \cdot \text{A1}$$

$$\text{Z012\_p4\_20} = \begin{pmatrix} 27.944 + 139.158i & -0.349 + 0.479i & 0.349 + 0.479i \\ 0.349 + 0.479i & 13.65 + 36.411i & 0.697 - 0.958i \\ -0.349 + 0.479i & -0.697 - 0.958i & 13.65 + 36.411i \end{pmatrix} \Omega$$

In lecture 10 we found:

$$\text{Dm} := (\text{Dab} \cdot \text{Dbc} \cdot \text{Dac})^{\frac{1}{3}} \qquad \text{Dm} = 9.857\,\text{ft}$$

$$\text{L1} := \text{Icons} \cdot \ln\left(\frac{\text{Dm}}{\text{GMR}}\right) \qquad \text{L1} = 1.932\,\frac{\text{mH}}{\text{mi}}$$

$$Z1 := (Rac + j\cdot 2\cdot \pi \cdot freq\cdot L1)\cdot Length \qquad Z1 = 13.65 + 36.411i\,ohm$$

$$Z012\_p4\_34 = \begin{pmatrix} 28.294 + 132.149i & -0.349 + 0.479i & 0.349 + 0.479i \\ 0.349 + 0.479i & 14 + 29.402i & 0.697 - 0.958i \\ -0.349 + 0.479i & -0.697 - 0.958i & 14 + 29.402i \end{pmatrix} \Omega$$

Similarly

$$GMRbundle := \sqrt{GMR3\cdot Da1a2} \qquad GMRbundle = 0.077\,ft$$

$$L1bun := Icons\cdot \ln\left(\frac{Dm}{GMRbundle}\right) \qquad L1bun = 1.56\,\frac{mH}{mi}$$

$$Z1bun := \left(\frac{Rac3}{2} + j\cdot 2\cdot \pi \cdot freq\cdot L1bun\right) Length \qquad Z1bun = 14 + 29.402i\,ohm$$

Consider an untransposed line described in problem 4.4 and Figure P4.4. Let the ground wire be 1/0 ACSR and recalculate the phase impedance matrix Zabc, the sequence impedance matrix Z012, and the unbalance factors. Compare with previous results from problem 4.20 for the same line without the ground wire.

Ground wire data:

$$Rac\_gw := 0.888\,\frac{ohm}{mi}$$ Table B.8, at 25C and 60Hz

$$GMR\_gw := .00446ft \qquad diameter\_gw := .398in$$

$$Rperlength\_gw := \begin{pmatrix} Rac + Rd & Rd & Rd & Rd \\ Rd & Rac + Rd & Rd & Rd \\ Rd & Rd & Rac + Rd & Rd \\ Rd & Rd & Rd & Rac\_gw + Rd \end{pmatrix}$$

$$Rperlength\_gw = \begin{pmatrix} 0.3683 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.3683 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.3683 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.9833 \end{pmatrix} \frac{ohm}{mi}$$

$Hgw = 47\,ft$ defined earlier

$Xgw := 0ft$ horizontal position is at our zero point

Now calculate distance between conductors (be careful of negative signs)

$$Dagw := \sqrt{(Hgw - Ha)^2 + (Xa - Xgw)^2} \qquad Dagw = 7.632\,ft$$

$$Dbgw := \sqrt{(Hgw - Hb)^2 + (Xb - Xgw)^2} \qquad Dbgw = 12.083\,ft$$

$$Dcgw := \sqrt{(Hgw - Hc)^2 + (Xc - Xgw)^2} \qquad Dcgw = 15.692\,ft$$

$$Lperlength\_gw := Icons \cdot \begin{pmatrix} \ln\left(\frac{De}{GMR}\right) & \ln\left(\frac{De}{Dab}\right) & \ln\left(\frac{De}{Dac}\right) & \ln\left(\frac{De}{Dagw}\right) \\ \ln\left(\frac{De}{Dab}\right) & \ln\left(\frac{De}{GMR}\right) & \ln\left(\frac{De}{Dbc}\right) & \ln\left(\frac{De}{Dbgw}\right) \\ \ln\left(\frac{De}{Dac}\right) & \ln\left(\frac{De}{Dbc}\right) & \ln\left(\frac{De}{GMR}\right) & \ln\left(\frac{De}{Dcgw}\right) \\ \ln\left(\frac{De}{Dagw}\right) & \ln\left(\frac{De}{Dbgw}\right) & \ln\left(\frac{De}{Dcgw}\right) & \ln\left(\frac{De}{GMR\_gw}\right) \end{pmatrix}$$

$$Lperlength\_gw = \begin{pmatrix} 3.749 & 1.81 & 1.874 & 1.899 \\ 1.81 & 3.749 & 1.766 & 1.751 \\ 1.874 & 1.766 & 3.749 & 1.667 \\ 1.899 & 1.751 & 1.667 & 4.296 \end{pmatrix} \frac{mH}{mi}$$

$$Zabc\_gw := (Rperlength\_gw + j \cdot 2 \cdot \pi \cdot freq \cdot Lperlength\_gw) \cdot Length$$

$$Zabc\_gw = \begin{pmatrix} 18.415 + 70.66i & 4.765 + 34.124i & 4.765 + 35.332i & 4.765 + 35.801i \\ 4.765 + 34.124i & 18.415 + 70.66i & 4.765 + 33.291i & 4.765 + 33.014i \\ 4.765 + 35.332i & 4.765 + 33.291i & 18.415 + 70.66i & 4.765 + 31.428i \\ 4.765 + 35.801i & 4.765 + 33.014i & 4.765 + 31.428i & 49.165 + 80.971i \end{pmatrix} \Omega$$

Now reduce this to an equivalent 3x3 matrix:

$Za := \text{submatrix}(Zabc\_gw, 0, 2, 0, 2)$

$Zb := \text{submatrix}(Zabc\_gw, 0, 2, 3, 3)$

$Zc := \text{submatrix}(Zabc\_gw, 3, 3, 0, 2)$

$Zd := \text{submatrix}(Zabc\_gw, 3, 3, 3, 3)$

$Zabceq := Za - Zb \cdot Zd^{-1} \cdot Zc$

$$Zabceq = \begin{pmatrix} 22.234 + 57.43i & 8.157 + 21.867i & 7.915 + 23.629i \\ 8.157 + 21.867i & 21.423 + 59.306i & 7.554 + 22.451i \\ 7.915 + 23.629i & 7.554 + 22.451i & 21 + 60.311i \end{pmatrix} \Omega$$

In problem 4.20 we found:

$$Zabc = \begin{pmatrix} 18.415 + 70.66i & 4.765 + 34.124i & 4.765 + 35.332i \\ 4.765 + 34.124i & 18.415 + 70.66i & 4.765 + 33.291i \\ 4.765 + 35.332i & 4.765 + 33.291i & 18.415 + 70.66i \end{pmatrix} \Omega$$

Comparing results:
(a) The resistance terms are larger, due to folding in the resistance of the ground wire
(b) The self inductance terms are smaller (in Xaa, Xbb, Xcc) since the phase conductors are coupling with a grounded conductor that is much closer than the earth.
(c) The inductive reactance of the off-diagonal terms are also smaller.

Now find the symmetrical components matrix:

$Z012\_p4\_40 := A1^{-1} \cdot Zabceq \cdot A1$

$$Z012\_p4\_40 = \begin{pmatrix} 37.303 + 104.314i & -0.297 - 0.886i & 1.3 - 0.502i \\ 1.3 - 0.502i & 13.677 + 36.367i & 0.747 - 0.973i \\ -0.297 - 0.886i & -0.707 - 1.009i & 13.677 + 36.367i \end{pmatrix} \Omega$$

and earlier we had:

$$Z012\_p4\_20 = \begin{pmatrix} 27.944 + 139.158i & -0.349 + 0.479i & 0.349 + 0.479i \\ 0.349 + 0.479i & 13.65 + 36.411i & 0.697 - 0.958i \\ -0.349 + 0.479i & -0.697 - 0.958i & 13.65 + 36.411i \end{pmatrix} \Omega$$

Comparing results:
(a) The resistance terms are only larger in the zero sequence terms, since the ground conductor only carries zero sequence currents
(b) The self inductance terms X1 and X2 are basically the same, but X0 is smaller for the case with the ground wire. Again, coupling to a closer ground conductor.
(c) The off-diagonal terms that couple the zero sequence show differences, those that couple positve to negative are essentially the same.
(d) So all of the difference seen in the Zabc matrices are basically in the zero sequence related terms.

Capacitance Matrix Calculations

$\varepsilon\_o := 8.854 \cdot 10^{-12} \frac{F}{m}$ $\qquad$ $Ccons := 2 \cdot \pi \cdot \varepsilon\_o$

$Haai := 2 \cdot Ha$ $\qquad$ $Haai = 81\,ft$ $\qquad$ $Hbbi := 2 \cdot Hb$ $\qquad$ $Hbbi = 72\,ft$

$Hcci := 2 \cdot Hc$ $\qquad$ $Hcci = 65\,ft$

$Habi := \sqrt{(Ha + Hb)^2 + (Xa - Xb)^2}$ $\qquad$ $Habi = 77.028\,ft$

$Haci := \sqrt{(Ha + Hc)^2 + (Xa - Xc)^2}$ $\qquad$ $Haci = 73.027\,ft$

$Hbci := \sqrt{(Hb + Hc)^2 + (Xb - Xc)^2}$ $\qquad$ $Hbci = 69.378\,ft$

GMR from table B.8

$diameter := 0.721in$ $\qquad$ $r := \frac{diameter}{2}$

$Length := 50mi$

$$P := \frac{1}{Ccons}\begin{pmatrix} \ln\left(\frac{Haai}{r}\right) & \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Haci}{Dac}\right) \\ \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Hbbi}{r}\right) & \ln\left(\frac{Hbci}{Dbc}\right) \\ \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hcci}{r}\right) \end{pmatrix} \qquad P = \begin{pmatrix} 0.142 & 0.037 & 0.039 \\ 0.037 & 0.14 & 0.032 \\ 0.039 & 0.032 & 0.138 \end{pmatrix}\frac{m}{pF}$$

$$Capperlength := P^{-1} \qquad Capperlength = \begin{pmatrix} 12.856 & -2.665 & -3.029 \\ -2.665 & 12.712 & -2.212 \\ -3.029 & -2.212 & 13.035 \end{pmatrix}\frac{nF}{mi}$$

$$Cabc := Capperlength \cdot Length \qquad Cabc = \begin{pmatrix} 642.815 & -133.226 & -151.454 \\ -133.226 & 635.583 & -110.594 \\ -151.454 & -110.594 & 651.753 \end{pmatrix} nF$$

4.34 Consider the line configuration shown in Figure P4.4. Instead of using a single conductor of 336,400 CM ACSR in each phase, with current carrying capacity of 530 amperes, suppose that each phase consists of a two-conductor bundle of two 3/0 ACSR conductors with capacity of 300 amperes/conductor. Let the two conductors of each bundle be seperated by 1.0ft vertically.

Compute the capacitance matrix Cabc for the bundled conductor configuration and compare with the previous solution (problem 4.20).

Since 4.20 neglects the ground wire, it will be neglected here too.

$$diameter3 := 0.502in \qquad r3 := \frac{diameter3}{2}$$

Spacing between conductors

Within the bundle:

$$Da1a2 := 1.0ft \qquad Db1b2 := 1.0ft \qquad Dc1c2 := 1.0ft$$

Calculate geometric mean radius of the bundle and use the 3x3 matrix method. The is an approximation of the 6x6 matrix approach.

$$Rsbundle := \sqrt{r3 \cdot Da1a2}$$

$$\text{Pequivbund} := \frac{1}{\text{Ccons}} \cdot \begin{pmatrix} \ln\left(\frac{\text{Haai}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{Habi}}{\text{Dab}}\right) & \ln\left(\frac{\text{Haci}}{\text{Dac}}\right) \\ \ln\left(\frac{\text{Habi}}{\text{Dab}}\right) & \ln\left(\frac{\text{Hbbi}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{Hbci}}{\text{Dbc}}\right) \\ \ln\left(\frac{\text{Haci}}{\text{Dac}}\right) & \ln\left(\frac{\text{Hbci}}{\text{Dbc}}\right) & \ln\left(\frac{\text{Hcci}}{\text{Rsbundle}}\right) \end{pmatrix}$$

$$\text{Cabc\_bu} := \text{Pequivbund}^{-1} \cdot \text{Length}$$

$$\text{Cabc\_bu} = \begin{pmatrix} 859.731 & -211.005 & -245.042 \\ -211.005 & 839.394 & -171.119 \\ -245.042 & -171.119 & 870.638 \end{pmatrix} \text{nF}$$

Now if we compare the original Cabc with the Cabc_bu:

$$\text{Cabc} = \begin{pmatrix} 642.815 & -133.226 & -151.454 \\ -133.226 & 635.583 & -110.594 \\ -151.454 & -110.594 & 651.753 \end{pmatrix} \text{nF}$$

Observations:
(1) The self-capacitance terms are larger with the bundled conductors, capacitance is proportional to the natural log of conductor diameter
(2) The off-diagonal capacitance terms also increase. However, if we looked at the offdiagonal terms in the P-matrices at the right, we see that they are nearly identical, so the differences in the capacitance matrices are from the inversion.

$$\text{C012\_p4\_20} := \text{A1}^{-1} \cdot \text{Cabc} \cdot \text{A1}$$

$$\text{C012\_p4\_34} := \text{A1}^{-1} \cdot \text{Cabc\_bu} \cdot \text{A1}$$

$$\text{C012\_p4\_20} = \begin{pmatrix} 379.868 & -10.866 - 0.594i & -10.866 + 0.594i \\ -10.866 + 0.594i & 775.141 & 20.88 + 15.192i \\ -10.866 - 0.594i & 20.88 - 15.192i & 775.141 \end{pmatrix} \text{nF}$$

$$\text{c\_1} := \left(\frac{2 \cdot \pi \cdot \varepsilon\_o}{\ln\left(\frac{\text{Dm}}{r}\right)} \right) \cdot \text{Length} \qquad \text{c\_1} = 772.69\,\text{nF}$$

Note that c_1 does not match the matrix term....

$$C012\_p4\_34 = \begin{pmatrix} 438.477 & -17.396 - 0.806i & -17.396 + 0.806i \\ -17.396 + 0.806i & 1065.643 & 39.508 + 28.671i \\ -17.396 - 0.806i & 39.508 - 28.671i & 1065.643 \end{pmatrix} \text{nF}$$

$$c\_1 := \left(\frac{2 \cdot \pi \cdot \varepsilon\_o}{\ln\left(\frac{Dm}{Rsbundle}\right)} \right) \cdot Length$$

$$c\_1 = 1060.322\,\text{nF}$$